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Resolution and predictability: An approach to the scaling problem

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Abstract

We analyzed the relationship between resolution and predictability and found that while increasing resolution provides more descriptive information about the patterns in data, it also increases the difficulty of accurately modeling those patterns. There are limits to the predictability of natural phenomenon at particular resolutions, and "fractal-like" rules determine how both "data" and "model" predictability change with resolution.

We analyzed land use data by resampling map data sets at several different spatial resolutions and measuring predictability at each. Spatial auto-predictability (P_a) is the reduction in uncertainty about the state of a pixel in a scene given knowledge of the state of adjacent pixels in that scene, and spatial cross-predictability (P_c) is the reduction in uncertainty about the state of a pixel in a scene given knowledge of the state of corresponding pixels in other scenes. P_a is a measure of the internal pattern in the data while P_c is a measure of the ability of some other "model" to represent that pattern.

We found a strong linear relationship between the log of P_a and the log of resolution (measured as the number of pixels per square kilometer). This fractal-like characteristic of "self-similarity" with decreasing resolution implies that predictability may be best described using a unitless dimension that summarizes how it changes with resolution. While P_a generally increases with increasing resolution (because more information is being included), P_c generally falls or remains stable (because it is easier to model aggregate results than fine grain ones). Thus one can define an "optimal" resolution for a particular modeling problem that balances the benefit in terms of increasing data predictability (P_a) as one'increases resolution, with the cost of decreasing model predictability (P_c).

Introduction

We hypothesized that an important determinant of the predictability of phenomenon is the scale (resolution and extent) of the analysis. By resolution we mean "grain size" or the size of the smallest unit of measure, with increasing resolution corresponding to fine grain. We can distinguish at least two ways that resolution might affect predictability. One is the increasing difficulty of building predictive models at increasingly finer resolution. For example, the position and velocity of individual molecules in a gas is highly unpredictable, but the temperature of the gas (which is an average of these motions at a much cruder resolution) is highly predictable. Likewise, it is easier to predict general climate patterns than it is to predict the exact geographic location and timing of rainstorms (the weather).

On the other hand, finer resolution allows more

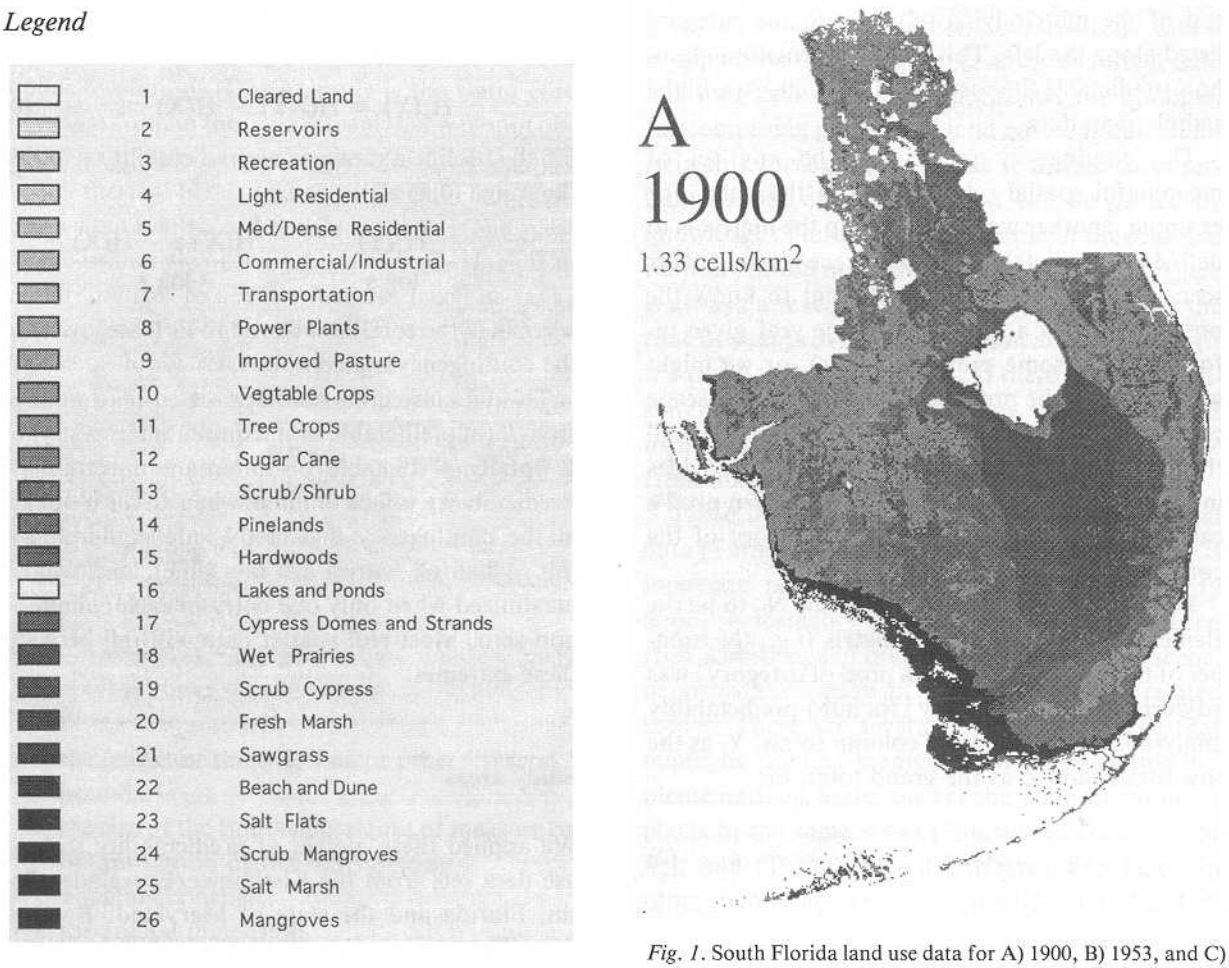


Fig. 1. South Florida land use data for A) 1900, B) 1953, and C) 1973 (from Costanza (1975)).

centuries. By 1953, the expanding US middle class and post-war economic boom had led to significant urban and agricultural development. This development accelerated into the 70s and 80s.

For the predictability analysis we used versions of the land use maps that had been manually digitized into 128 acre (0.52 km^2) rectangular cells, each 625 m by 833 m (Costanza 1979). This produced an array with overall dimensions of 576 rows by 400 columns (230,400 total cells) of which about 93,000 were inside the study area. Each cell was assigned one of 26 land use categories ranging from natural to agricultural to urban systems (Costanza 1979).

State of Maryland

The Maryland Department of State Planning has compiled digitized land use data (using 80 acre cells) for three different years (1973, 1981, and 1985) for the entire state, using 20 different land use categories. While there were significant shifts in land use in Maryland, land use had changed much less dramatically in this area over the 22-year interval between 1973 and 1985 than it had in South Florida during the 1900 to 1973 interval. Major changes involved reforestation of agricultural areas and significant increases in urban land uses, especially in the Washington/Baltimore corridor. For the state of Maryland we used a rectangular array of 345 rows by 640 columns or a total of 220,800 cells, of

detail to be observed and internal patterns in the data to be seen that may not have been observed at cruder resolutions. One example is the warm core gyres that form in the Gulf Stream that were not observed until remote sensing images including the proper thermal bands and of sufficiently fine resolution were available. Another example is the quest by the military to obtain high enough resolution satellite images to see the features (such as tanks and airplanes) of interest to them that would not appear on lower resolution images.

Some phenomenon are known to vary in a regular way with resolution. For example, the regular relationship between the measured length of a coastline and the resolution at which it is measured is a fundamental one behind the concept of fractals (Mandelbrot **1977)** and can be summarized in the following equation:

$$L = k\, s^{(1-D)} \qquad (1)$$

where:

L = the length of the coastline or other "fractal" boundary
s = the size of the fundamental unit of measure or the resolution of the measurement
k = a scaling constant
D = the fractal dimension

This convenient "scaling rule" has proved to be very useful in describing many kinds of complex boundaries and behaviors (Mandelbrot **1983,** Milne **1988,** Turner ***et al.*** **1987, 1989,** Olsen and Schaffer **1990,** Sugihara and May **1990).** We hypothesized that this same kind of relationship might exist between resolution and predictability (and possibly other measures as well) and might be useful for developing scaling rules for understanding and modeling. We tested this hypothesis by calculating both data and model predictability for a number of landscapes at a number of different resolutions.

Measurement of predictability

Colwell **(1974)** applied information theoretic concepts to the problem of estimating the degree of predictability of periodic phenomena. The method is similar to autocorrelation analysis except that it is applicable to both interval and categorical data and may thus be more appropriate, for example, for comparing patterns of land cover. Predictability in this context refers to the reduction in uncertainty about one variable that can be gained by knowledge of another. For example, if the seasonal rainfall pattern in an area is predictable ***(e.g.,*** there is always a severe dry summer), then knowing the time of year provides information about rainfall (if it's summer, it must be dry). If there is no relationship between rainfall and season, time of year tells us little and the rainfall is relatively unpredictable from a knowledge of time of year.

These techniques can also be applied to spatial data (Turner ***et al.*** **1989).** In this application, one is interested in the degree to which the uncertainty about the category of a particular pixel is reduced from knowledge of other aspects of the same scene, or from knowledge of aspects of other, related scenes. There are several aspects of a scene that might be used as predictors. We discuss two implementations based on **(1)** the state of adjacent pixels in the same scene ("auto-predictability" or P_a); and (2) the state of corresponding pixels in other, related scenes ("cross predictability" or P_c). Other combinations of these two and higher level analyses ***(i.e.,*** adjacent pixel pairs, triplets, etc., or multiple cross comparisons) are also possible and useful for various purposes (Turner ***et al.*** **1989).**

The method in general can determine if there are regularities in a spatial data set, ranked on a scale from 0 (totally unpredictable) to **1** (totally predictable), and the answer can be interpreted as the degree of departure of the scene or comparison between scenes from a random (totally unpredictable) pattern.

To estimate predictability, one first assembles a contingency matrix with states or conditions of the pixels along the left axis, and corresponding states of other pixels along the top. For auto-predictability the categories in a map are listed on the left and along the top of a matrix. The numbers in the matrices represent the frequency of occurrence in the mapped data of the category (or category pair, triplet, etc. for higher level analysis) listed along the

top of the matrix lying adjacent to the category listed along the left. This yields information about how predictable the patterns of adjacency are in the sample map data.

The contingency matrix can be any set of meaningful spatial relationships in the data. For example, another way of setting up the matrix is to define the predictability of one scene given another scene. For example, we might want to know the predictability of a landscape in one year given information in some previous year(s), or we might want to know the predictability of a real landscape compared to a landscape model's output. We call this the "cross" predictability, because it provides information on the predictability of a given pixel's category given knowledge of the category of the corresponding pixel in another scene.

Following Colwell (1974) we define N_{ij} to be the elements in the contingency matrix (*i.e.*, the number of times in the data that a pixel of category i was adjacent to one of category j for auto-predictability analysis). Define X_j as the column totals, Y_i as the row totals, and Z as the grand total, or:

$$X_j = \sum_{i=1}^{s} N_{ij} \quad (2)$$

$$Y_i = \sum_{j=1}^{t} N_{ij} \quad (3)$$

and

$$Z = \sum_i \sum_j N_{ij} = \sum_j X_j = \sum_i Y_i \quad (4)$$

Then the uncertainty with respect to **X** is:

$$H(X) = -\sum_{j=1}^{t} \frac{X_j}{Z} \log \frac{X_j}{Z} \quad (5)$$

and the uncertainty with respect to **Y** is:

$$H(Y) = -\sum_{i=1}^{S} \frac{Y_i}{Z} \log \frac{Y_i}{Z} \quad (6)$$

and the uncertainty with respect to the interaction of **X** and **Y** is:

$$H(XY) = -\sum_i \sum_j \frac{N_{ij}}{Z} \log \frac{N_{ij}}{Z} \quad (7)$$

Then define the conditional uncertainty with regard to **Y** with **X** given as:

$$H_x(Y) = H(XY) - H(X) \quad (8)$$

Finally, define a measure of predictability (P) with the range (0,1) as:

$$P = 1 - \frac{H_x(Y)}{\log s} = 1 - \frac{H(XY) - H(X)}{\log s} \quad (9)$$

where s is the total number of rows (categories)-in the contingency matrix.

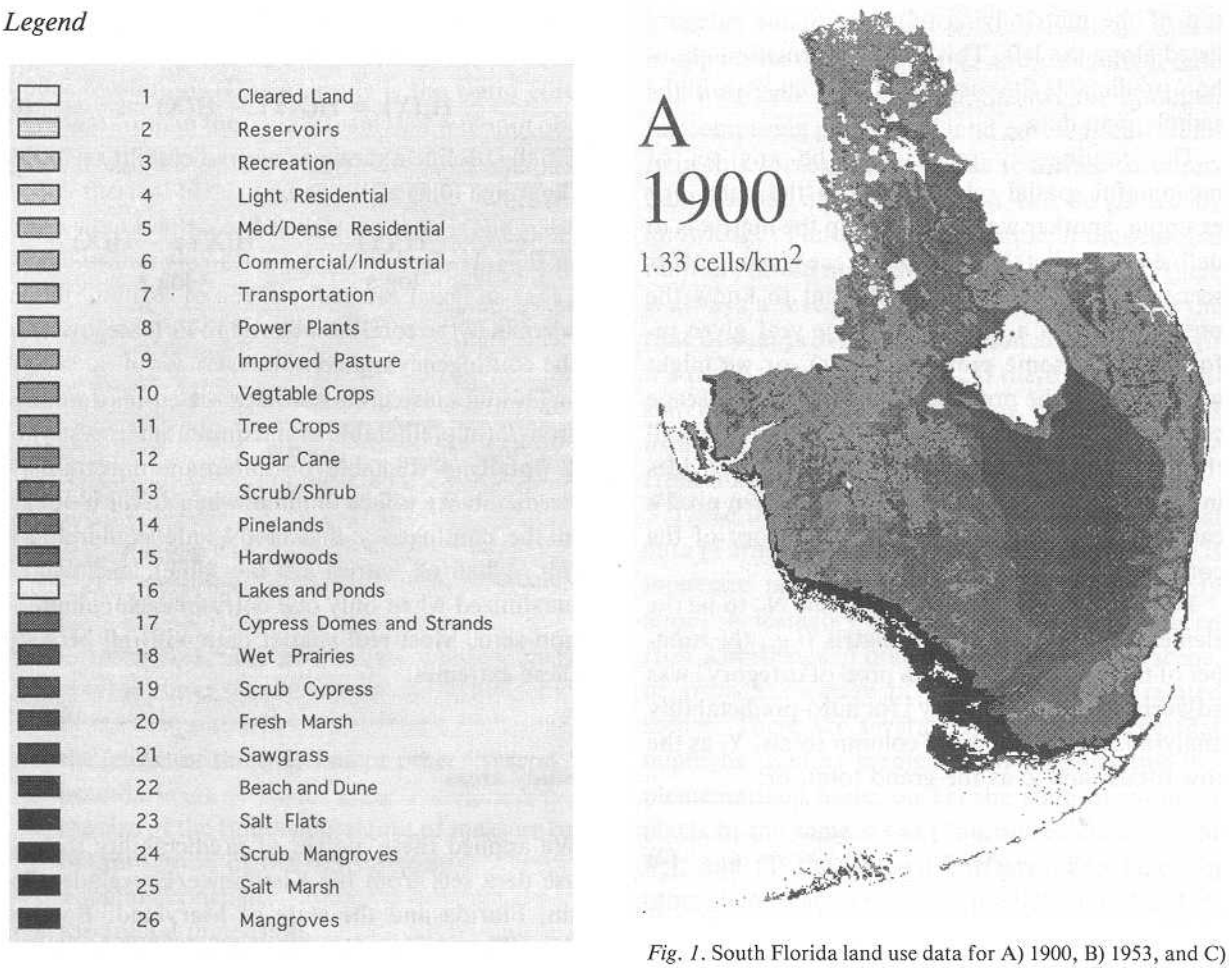
This measure gives an index scaled on the range from *0* (unpredictable or maximum uncertainty) to 1 (totally predictable or minimum uncertainty). Predictability will be minimal when all the elements in the contingency matrix (N_{ij} are equiprobable (*i.e.*, when all entries are the same), and will be maximized when only one entry in each column is non-zero. Most real spatial data will fall between these extremes.

Study areas

We applied these indices of predictability to land use data sets from the Kissimmee/Everglades Basin, Florida and the state of Maryland. Both of these data sets contained three distinct years of data over which significant changes in land use patterns had occurred.

Kissimmee/Everglades Basin, Florida

The Kissimmee/Everglades drainage basin in South Florida represents one of the most rapidly changing and intensively modified landscapes in the country. It consists of some 18,700 square miles (48,500 km^2) of land and water (Fig. 1) covering about 1/4 of the state of Florida. **A** set of three land use maps with *26* land use categories were prepared for the years 1900, 1953, and 1973 in order to analyze the dramatic changes that had occurred in the region during this interval (Costanza 1975,1979). In 1900, when much of the United States had already been developed into farmland and cities, the Kissimmee/Everglades basin remained much as it had been for



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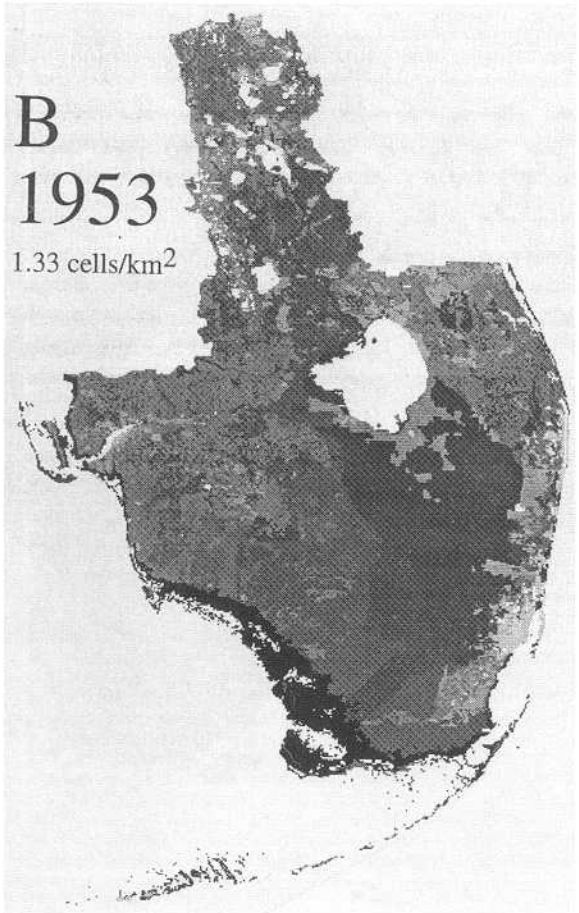

Fig. 1B.

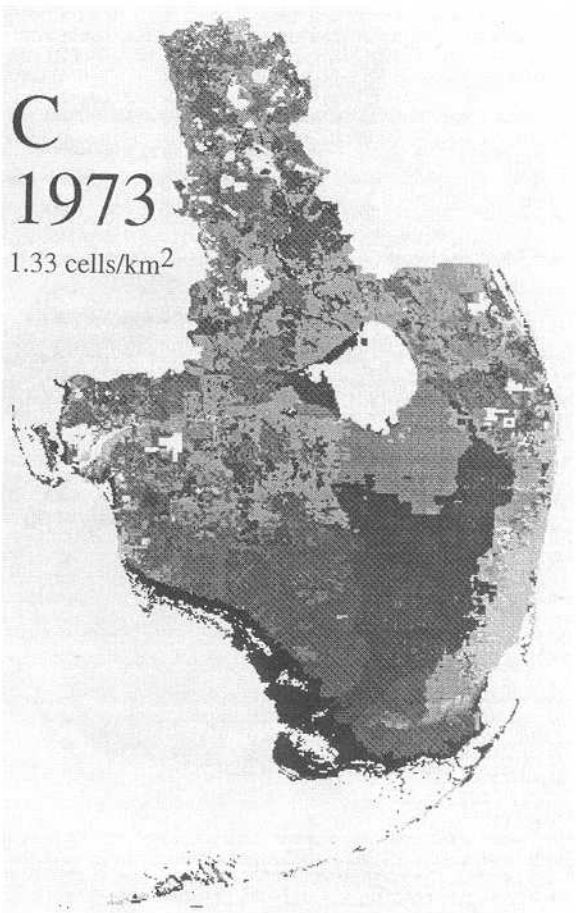

Fig. 1C.

which approximately 68,800 were within the boundaries of the state.

Software and algorithms

Land use data from the two study sites was imported into Map II™, a simple and easy to use raster-based GIS package for the Macintosh, to prepare data files for calculations using high speed parallel transputers, as discussed below. Any GIS system capable of producing raster output would be suitable, however.

Decreasing the resolution (increasing the grain) of a spatial data set involves the repetitive resampling of a specified number of small cells into larger cells. Analytically, this is accomplished by moving a resampling matrix (whose size is the number of rows and columns of the original data needed to make a single cell in the new raster) through the original raster. The cell values falling within the resampling matrix are tabulated and used to determine the value of the appropriate larger cell in the new, coarser resolution raster. We experimented with several methods of resampling or aggregating the spatial data. The first method, which we call *proportional aggregation*, assigns the cell values in the coarser grain raster according to the most dominant category found within the resampling matrix. A second method, termed *random aggregation*, assigns new categories by randomly choosing from the categories found within the resampling matrix. The major difference between the two methods is that rare categories are more likely to be preserved when the data are resampled with the random aggregation scheme. While the choice of aggrega-

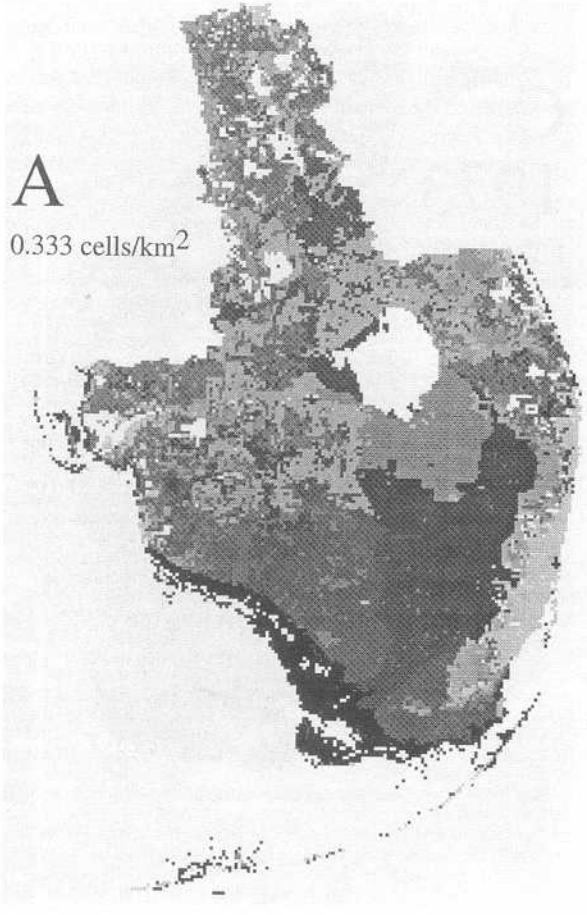

Fig. 2. Example of the random, sequential aggregation scheme applied to the 1973 South Florida data four successive times. These four aggregations, along with the original, make up the five different resolutions used in the analyses. The resolutions used for the Florida site were 1.333 (original), 0.333 (A), 0.083 (B), 0.021 (C), and 0.005 (D) cells/km^2.

tion scheme can be significant in many spatial analyses, we found that the aggregation scheme made little difference to the results of our particular experiments. We settled on a version of the random aggregation scheme that is both simple and suits our immediate needs. In this version aggregation takes place in steps. In each step the original map is aggregated using a 2×2 resampling matrix, yielding an aggregated map with 1/4 the number of cells of the original. In each 2×2 resampling matrix we choose the category of the northwest cell as the category for the cell in the aggregate map. This process was repeated on the new aggregate map to yield a series of maps each with 1/4 the total number of

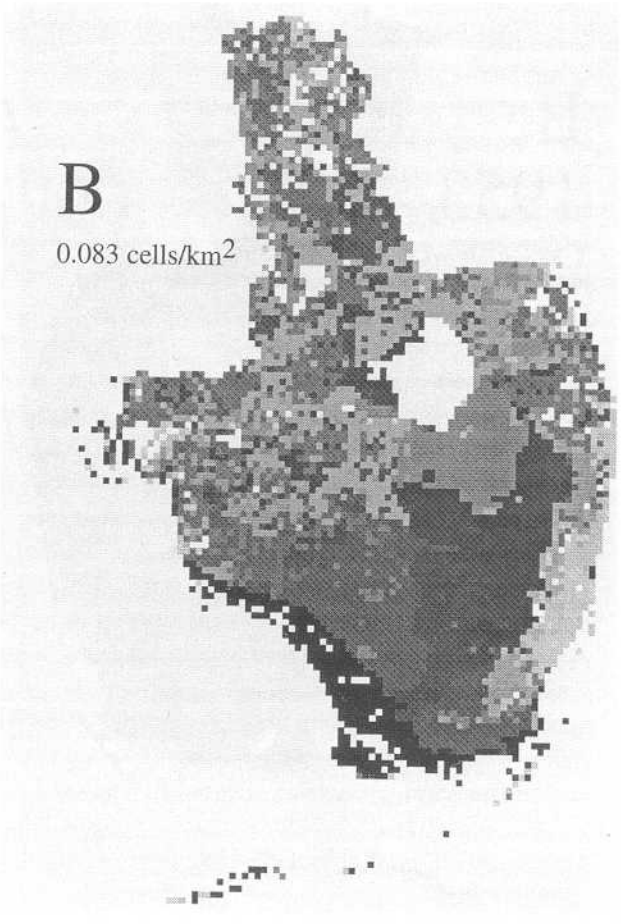

Fig. 2B.

cells of the one preceding it in the series. Figure 2 shows the results of this process for the South Florida, 1973 data set. Each map in this series has 1/4 the resolution of the preceding map. For the Florida maps the resulting resolutions (in cells/km^2) were 1.333 (original), 0.333, 0.083, 0.021, and 0.005. For the Maryland maps the resulting resolutions (in cells/km^2) were 2.743 (original), 0.686, 0.171, 0.043, and 0.011.

We developed algorithms in a parallel version of the C programming language to calculate auto and cross-predictability for mapped data on Inmos Transputers (a form of RISC based parallel processor) on a Macintosh (Costanza and Maxwell 1991). Transputers are extremely fast for this sort of calculation. For example, for the South Florida data (a 576×400 array) calculation of auto-predictability and printing results to a text file took approximate-

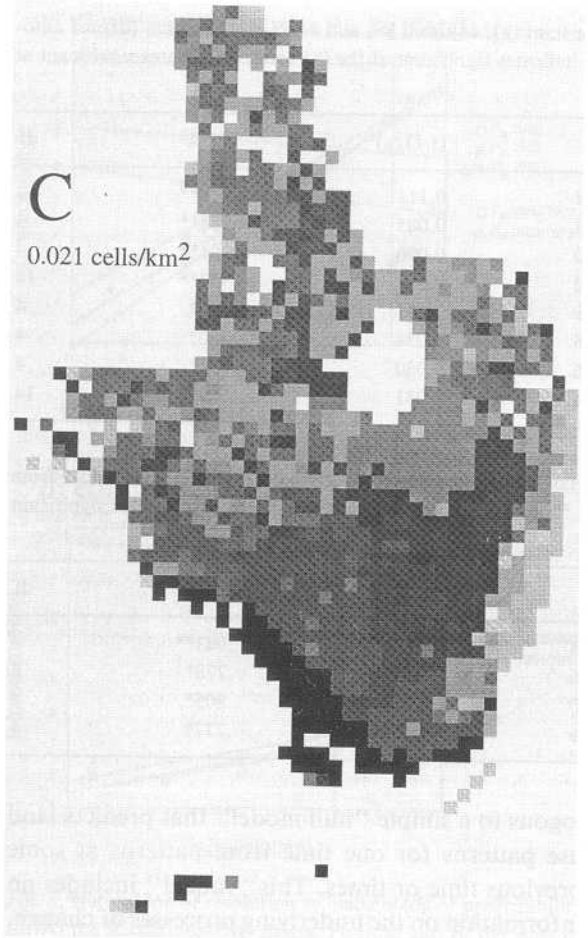

Fig. 2C.

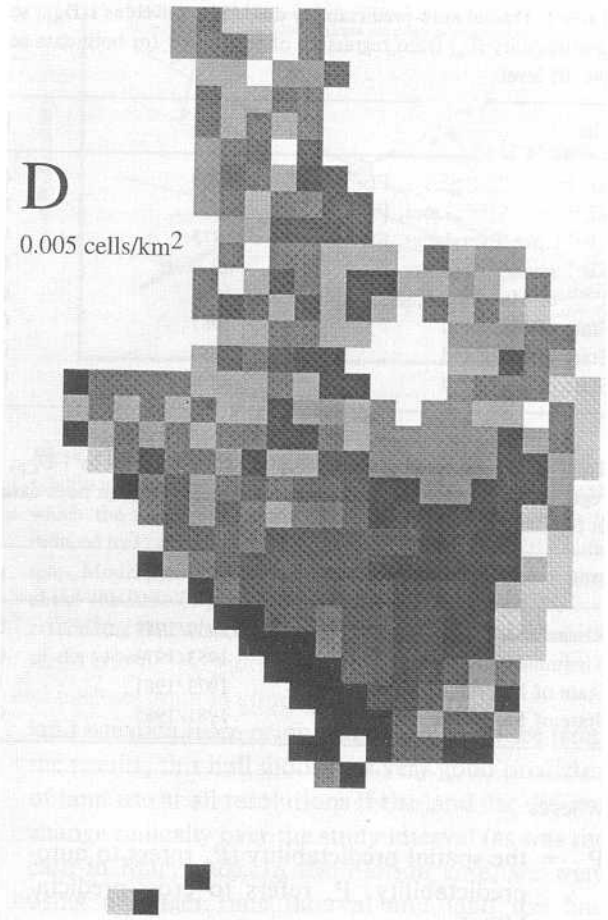

Fig. 2D.

ly 2.4 seconds using a Macintosh IIci with 4 transputers[1].

Auto-predictability experiments

We calculated P_a for both study areas for all three years and at five different resolutions. We started with the maximum resolution of the data and gradually degraded it by aggregating pixels. In each step we halved the resolution by aggregating 2×2 blocks of pixels at the previous resolution into single pixels as described above. Resolution is frequently indicated as the length of a side of a cell (pixel), with higher or finer resolution corresponding to smaller cell (pixel) sizes. For example, LANDSAT satellite data has 30-meter resolution while SPOT satellite data has finer resolution at 18 meters. In our plots we wanted higher resolution to correspond to higher (not lower) numbers so we measured resolution as the number of cells per km^2. For example, 50-meter cells would have a resolution of 400 cells/km^2, while 200-meter cells would have a resolution of 25 cells/km^2.

We fit the equation:

$$P = k\, r^{(1-D_P)} \qquad (2)$$

[1]The algorithms also work on serial machines, only slower. Each transputer is approximately the speed of a SUN Sparc station so the 4 transputer time is about four times the speed one would expect on a Sparc station. Contact Tom Maxwell for more information about obtaining the algorithms or using transputers for spatial analysis.

Table 1. Fractal auto-predictability dimension (given as $1\text{-}D_{AP}$, scale constant (k), adjusted R^2, and degrees of freedom (df) for auto-predictability (P_a) from regression of equation 3 for both data sets. ** indicates significant at the **0.1** level, * indicates significant at the .05 level.

Site	Year	k	$(1\text{-}D_{AP})$	adj R^2	df
Kissimmee/Everglades, FL	1900	0.6364	0.111	.999**	4
Kissimmee/Everglades, FL	1953	0.6383	0.085	.988**	4
Kissimmee/Everglades, FL	1973	0.6250	0.096	.981**	4
Kissimmee/Everglades, FL	all years	0.6332	0.097	.958**	14
State of Maryland	1973	0.5189	0.031	.780*	4
State of Maryland	1981	0.5046	0.034	.780*	4
State of Maryland	1985	0.4956	0.030	.631*	4
State of Maryland	all years	0.5434	0.031	.720**	14

Table 2. Fractal cross-predictability dimension (stated as $1\text{-}D_{CP}$, scale constant (k), adjusted R^2, and degrees of freedom (df) from regression of equation 3 for cross-predictability (P_c) for both data sets. ** indicates significant at the 0.1 level, * indicates significant at the .05 level.

Site	Year	k	$(1\text{-}D_{CP})$	adj R^2	df
Kissimmee/Everglades, FL	1900/1953	0.5764	−0.11	.943**	4
Kissimmee/Everglades, FL	1953/1973	0.4936	−.017	.778*	4
State of Maryland	1973/1981	1.0790	−.006	.805*	4
State of Maryland	1981/1985	0.9296	−.004	.777*	4

where:

- P = the spatial predictability (P_a refers to auto-predictability, P_c refers to cross-predictability)
- r = the resolution measured as the number of cells/km^2
- **k** = a scaling factor
- D_P = the fractal predictability dimension (dimensionless)

by first transforming it into log-log form:

$$ln\,(P_1) = ln\,(\mathbf{k}) + (1 - D_P) ln\,(r) \qquad (3)$$

and using standard linear regression analysis to solve for the parameters **k** and D_P.

The results are summarized in Table 1, which indicates the high R^2 for this relationship for both of the study sites.

Cross-predictability experiments

We calculated P, for both of the study areas by comparing maps from different years. This is analogous to a simple "null model" that predicts land use patterns for one time from patterns at some previous time or times. This "model" includes no information on the underlying processes of change, but we were interested in how changing the resolution of the maps affected the predictability, and the "null model'' of no change is an interesting point of reference. We fit equation **3** to the data and the results for the three sites are shown in Table 2.

Results of both the auto and cross-predictability experiments for both sites are plotted together on a log-log scale in Fig. **3.** The strong linearity of the relationship for all cases is apparent, as is the fact that auto-predictability (P_a) increases with increasing resolution while cross-predictability (P_c) decreases slightly with increasing resolution, although with a smaller D_P. These results are consistent with our original hypotheses. Results for the Kissimmee/Everglades data are markedly different from those for the Maryland data. The auto-predictability of Maryland land use changed much less with resolution than the Kissimmee/Everglades land use. The slope of the regression line ($1\text{-}D_{AP}$ for the Kissimmee/Everglades data was roughly three

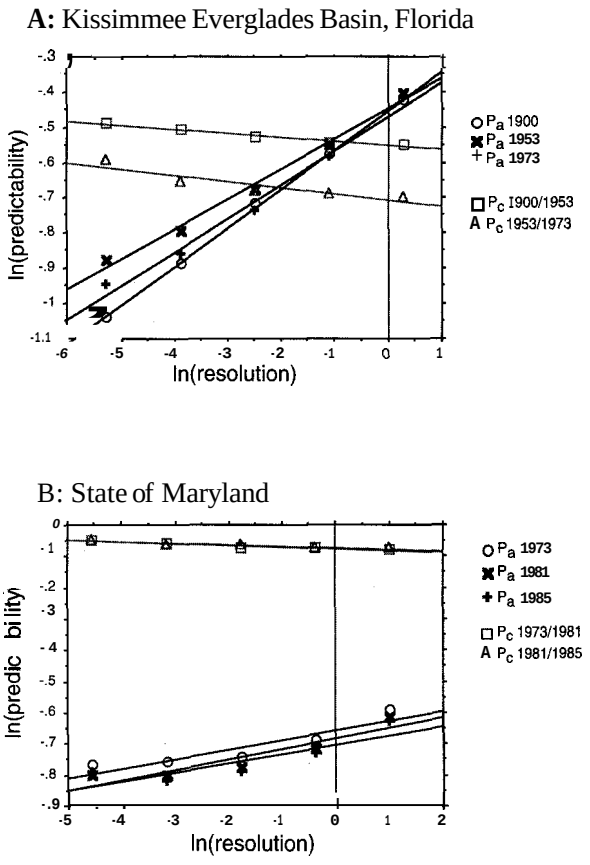

Fig. 3. Natural log **of** resolution vs. natural log of predictability for **A)** the Kississmee/Everglades, Florida, and **B)** the state of Maryland land use data. Plot shows both auto-predictability (P_a) indicating internal pattern in the data for three different years, and cross-predictability (P,) indicating pattern matching between null models of prior land use maps and the particular map. The resolutions used (in cells/km^2) were: Florida: 0.005, 0.021, 0.083, 0.333, 1.333; Maryland: 0.011, 0.043, 0.171, 0.686, 2.743.

times that for the Maryland data. Auto-predictability varied from about **0.65** to 0.35 over the range of resolutions used for the Florida data, but only from about **0.55** to 0.45 for the Maryland data. The Kissimmee/Everglades data was also more predictable at the highest resolutions than the Maryland data.

The cross-predictability results also differ markedly between the Florida and Maryland data. The slope of the regression line ($1 - D_{cp}$ was about three times higher for the Florida data than the Maryland data. Recall that the "null models" we are comparing with "data" in this analysis were

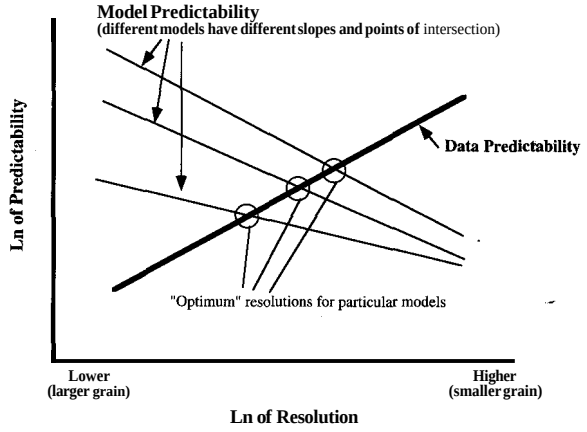

Fig. 4. Hypothetical relationship between resolution and predictability of data and models. Data predictibility is the degree to which the uncertainty about the state of landscape pixels is reduced by knowledge of the state of adjacent pixels in the same map. Model predictability is the degree to which the uncertainty about the state of pixels is reduced by knowledge of the corresponding state of pixels in output maps from various models of the system.

land use data from prior years. **As** we can see from the results, this null model is a very good predictor of land use at all resolutions if the land use did not change radically over the study interval (as was the case in Maryland). In the Florida case, we were using a longer time interval and land use had changed radically over this interval, so the null model was much less accurate at all resolutions.

In addition, this "null model" is of limited real usefulness since it embodies none of the underlying processes that caused the land use changes in the first place. In the more general case of dynamic landscape models, or models in general, we would not expect such high initial values of predictability, and would expect the predictability to fall more quickly with resolution. We are currently building dynamic landscape models to test this hypothesis which can be summarized in Fig. 4.

Discussion and conclusions

We can draw several conclusions from our analysis:

1. P_a and P_c belong to a class of "fractal-like" self-similar measures that vary in a regular way as resolution changes. This allows a "fractal

dimension" to be calculated that permits easy conversion of measurements of P taken at one resolution to other resolutions (for example, resolutions higher than those for which we have data). We suspect that there are many other spatial measures, which also exhibit this kind of self-similarity, that may be useful in developing a generalized theory of scaling.

2. P_a generally increases with increasing resolution. This relationship represents the "benefit" in terms of information gained about the pattern as resolution is increased.
3. P_c generally falls with increasing resolution. This relationship represents the "cost" of decreased model predictability as resolution is increased.
4. Combining 2 and 3 leads to some hypotheses about determining an "optimal" resolution for specific studies. At very low resolution it is easier to build predictive models, but they have little useful detail. At high resolution much useful detail is retained, but models are less able to predict it. An optimal resolution for scientific analysis may occur where these trends intersect – where one is balancing the costs and benefits incurred with increasing resolution. These results are consistent with empirical data from a survey of over *85* models of freshwater wetlands (Costanza and Sklar 1985).
5. The "models" we have analyzed so far are very primitive "null models" that one would expect to be different in overall predictability (P,) and in their fractal predictability dimensions (D_{cp}) than more sophisticated process-based spatial models (Costanza *et al.* 1990). We suspect that there is a different optimal resolution for each class of models, and possibly for each particular set of modeling objectives. We also suspect that P_c and its associated D_{cp} will change with changing technology and modeling skills. We are currently pursuing research aimed at addressing these questions by applying process-based spatial models at several different resolutions.
6. These results may be generalizable to all forms of resolutions (spatial, temporal, and number of components) and may shed some interesting light on "chaotic" behavior in systems. When looking across resolutions, chaos may be the low level of model predictability that occurs as a natural consequence of high resolution. Lowering model resolution can increase model predictability by averaging out some of the chaotic behavior at the expense of losing detail about the phenomenon. For example, Sugihara and May (1990) found chaotic dynamics for measles epidemics at the level of individual cities, but more predictable periodic dynamics for whole nations. The idea is not to maximize the resolution of analysis in order to "discover" this "unpredictable" chaotic behavior, nor is it to maximize predictability by ignoring details. Rather, the aim is to choose the resolution that maximizes the *effectiveness* of the model in balancing the conflicting trends of data and model predictability with changing resolution.

Acknowledgments

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