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Energy Intensities, Interdependence, and Value in Ecological Systems: A Linear Programming Approach

ROBERT COSTANZA AND CHRISTOPHER NEILL

Coastal Ecology Laboratory, Center for Wetland Resources, Louisiana State University, Baton Rouge, Louisiana 70803, *U.S.A.*

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Many authors have applied input–output techniques to the analysis of energy and material flows in ecosystems. These applications are important contributions to our understanding of interdependence in ecological systems, but they suffer from reliance on "single commodity" flow matrices (i.e. enthalpy or carbon) and they do not address the problem of joint products, which is particularly significant for ecological applications. In this paper a general linear programming model is developed and used to describe and analyze interdependence in multicommodity ecosystems with joint products. The model is applied to data for Silver Springs and embodied energy intensities for the ecosystem products are calculated. The effects of different assumptions concerning the treatment of waste heat and joint products on the energy intensities are also discussed, as are potential applications to resource management problems that require a measure of relative value.

1. Introduction

The concept of value can play a major role in both ecology and economics. Both fields are involved in the study of the complex interdependence between physically dissimilar components. The concept of value allows comparisons to be made between these dissimilar components. The "valuation problem" for any system is therefore analogous to answering the question: what weighting or conversion factors can be used to compare essentially dissimilar, non-commensurable items, such as cars, swamps, and eagles? Is there a common denominator useful in linking all system components and making the cost/benefit calculations necessary to predict surviveability?

The weighting factors in modern economic systems are prices. Several alternatives have been proposed as the analog of prices for ecological systems. The most common is calorimetric energy content (Odum, 1971). Any proposed valuation or weighting system must satisfy the basic accounting constraint: the sum of the values of inputs (weighting factors times

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quantities) must equal the sum of values of outputs for all transformation processes. This conservation constraint holds in dollars for economic entities, for calorimetric energy (by the first law of thermodynamics), and for mass (more accurately, for the total mass and energy). However, the use of first law accounting in ecology does not guarantee a "value" solution analogous with prices. The problem with first law accounting is the existence of the second law of thermodynamics, which states that the "usefulness" or availability of calorimetric energy decreases in any transformation. As Georgescu-Roegen (1971) has pointed out, it is low-entropy matter and energy that are "useful" and therefore necessary for economic value. An accounting that recognizes the second law is more appropriate for ecological and economic systems, since it allows value calculations that take into account usefulness or availability. It is the purpose of this paper to extend the development of such an accounting.

One form of second law accounting assumes, like all of classical thermodynamics, that all processes are reversible (Clapp, 1981; Ross, 1978). Reversible processes occur infinitely slowly and at maximum (Carnot) efficiency. Processes in the real world occur at finite rates, however, and are irreversible. Useful accountings of energy flows in ecological systems must therefore incorporate concepts of time, irreversibility, and power (Odum & Pinkerton, 1955; Prigogine, 1980). Accounting systems that do this are called irreversible second law accountings.

In irreversible systems the notion of "dissipative structures" is important (Prigogine, 1980). A dissipative structure is one that maintains its own ordered structure by dissipating gradients elsewhere in the system. The concept of "value" is hypothesized to correspond most closely with "degree of organization." Quantifying the degree of organization of dissipative structures (like cars, swamps, and eagles) is no simple matter. One approach, through information theory, is to estimate the degree of departure of the structure from a random arrangement of its elements (Gatlin, 1972; Ulanowicz, 1980; Jorgensen, 1982). While promising, this approach is not yet fully operational for complex structures, because the data and computations necessary to investigate all possible levels of organization are formidable.

Since creating and maintaining organized structures requires the dissipation of energy, a second approach to estimating the degree of organization in real, irreversible systems is to calculate the calorimetric energy required directly and indirectly to produce the structure. This dissipated energy is then said to be "embodied" in the organized structure (Costanza, 1980). Embodied energy serves as an index of order in the same sense that measuring the energy required to pump water into a reservoir is an index of the available energy stored in the elevated water. Embodied energy has also been shown to correlate well with economic value given appropriate assumptions (Costanza, 1980). Using this approach we can calculate theoretically the energy embodied in the structure of plants, animals, soil, water, and all other ecological "goods and services" once we know the detailed web of transformations that lead to their production.

One approach to this complex calculation is to use systems of linear equations to represent the transformations, and the well-known mathematical techniques of input–output analysis to solve them. This has been attempted by Hannon (1973, 1976, 1979) for selected ecosystems but, as Herendeen (1980) points out, the results were somewhat confusing. We will show that Hannon was really performing a first law accounting and will go on to demonstrate how an irreversible second law accounting that allows for joint products may be performed using the Silver Springs, Florida, ecosystem data originally published by Odum (1957).

2. Joint Products

Joint products refer to the interdependent by-products of most real transformation processes. For the purposes of discussion we can divide all production processes into two fundamental types: sorting and assembly. A production process is defined here as one that increases the degree of organization of some of its "products" while dissipating the order of at least some of its inputs. Sorting is a fundamental anti-entropic activity involving the splitting of a mixed substance into its components; assembly involves combining inputs into an organized product with lower entropy. If all production processes of interest were of the pure assembly type, then joint products might not present a problem, but the existence of sorting implies that joint products are unavoidable. All industries in the U.S. economy produce joint products. The mining industry, for example, produces refined minerals but also spent ore. The pulp produced by the timber industry and the sulfur removed from crude oil during refining are examples of joint products that are marketable. Other joint products are considered waste, such as air and water pollutants, and have no market value.

All ecological processes also produce joint products. Trees produce oxygen, water vapor, and waste heat as well as tree biomass. Waste heat is the most ubiquitous of joint products and requires special treatment, as discussed below. Although joint products seem to reflect the physical nature of economic and ecological processes (it is impossible to produce iron without also producing spent iron ore or to produce a tree without producing oxygen), they complicate mathematical description and analysis and have often been ignored.

Waste heat is an unavoidable by-product of all transformation processes. To understand its special role we must define a third fundamental process already alluded to—dissipation of an existing gradient. Dissipation is not a production process (since it proceeds spontaneously in the direction of increasing entropy) but is required to drive all production processes (which produce low entropy structures). The second law of thermodynamics requires that the amount of disorder produced by dissipation be greater than or equal to the amount of order created by sorting and assembly for irreversible processes in an isolated system. To sort or assemble at a finite rate, fuel, capital, labor, etc. must be "consumed" or "dissipated", by allowing the gradients to dissipate in a controlled way at a finite rate (cf. Odum & Pinkerton, 1955; Prigogine, Allen & Herman, 1977).

We can thus divide joint products into two distinct groups depending on their origin as the result of sorting or dissipation. This distinction is critical because the products of a purely dissipative process have no order and therefore no ability to perform work, no available embodied energy and no "value," while the products of sorting do.

For example, a temperature gradient could be allowed to dissipate without doing any work, in which case the only product would be uniform heat. The available energy embodied in the original gradient would disappear. On the other hand, we could insert an engine between the high temperature source and low temperature sink, and the available energy embodied in the original temperature gradient could be used to produce another gradient (sorting or assembly "work" via the engine). We could say that the dissipation of the original gradient was a necessary cost of production, and the original gradient could be said to be "embodied" in the products of the engine.

In real, irreversible systems, gradients are frequently not dissipated completely. For example, the exhaust from an automobile engine is above environmental temperatures and thus still has some available energy. It is immediately dissipated and lost without doing any work when the exhaust is released to the environment. Since this is a necessary cost of driving the production process at a finite rate, we might conclude that the energy embodied in the entire gradient should be embodied in the outputs. Alternatively, we could reason that only the gradient between the engine combustion temperature and the exhaust temperature be embodied in the outputs. We will discuss the implications of these alternatives on the resulting calculations.

3. Methods

The problem is to trace the flow of a primary resource (energy) through a complex web of multicommodity biological and chemical transformations to determine the amount required directly and indirectly to produce each of the components of an ecological system. This problem is analogous to the study of interdependence and the imputation of value to primary factors in economics. The economic problem was first explicitly formulated by Walras (1896) and empirical applications began with Leontief's (1941) Nobel prize-winning work. Leontief's approach has come to be known as input–output (I–O) analysis. I–O analysis has been applied to the study of energy and material flow in ecosystems (Hannon, 1973; Richey *et al.*, 1978) and in economic systems (Bullard & Herendeen, 1975; Costanza, 1980) and we refer the reader to these works for more detailed descriptions of the technique. I–O analysis can be shown to be a special case of the more general Linear Programming (LP) analysis (Dorfman, Samuelson & Solow, 1958). I–O analysis results when two simplifying assumptions are imposed on the LP model: (1) no joint products; (2) no alternative production technologies (i.e., there is a unique combination of inputs for each output).

Since it is just these two assumptions that we wish to relax, the LP approach provides the appropriate analytical framework. We proceed by developing the LP model, noting correspondence with the I–O model where appropriate.

A general equilibrium model of an economy that allows joint products and alternative production technologies was first proposed by von Neumann (1945). In the von Neumann model there are m commodities (denoted by $C_1, C_2, \ldots, C_m$) and n processes or activities for transforming commodities (denoted by $Q_1, Q_2, \ldots, Q_n$). Each transformation process can be indicated by its input commodity requirements and its jointly produced outputs, for example for process j:

$$Q_j = \begin{bmatrix} U_{1j}, U_{2j}, \ldots, U_{nj} \\ V_{j1}, V_{j2}, \ldots, V_{jn} \end{bmatrix}$$

or (U_{ij}, V_{ji}), $i=1, m$; $j=1, n$, where U_{ij} = the input of commodity i to process j, V_{ji} = the output of commodity i from process j and one discrete time period or accounting unit is implied. Figure 1 illustrates this general production relation.

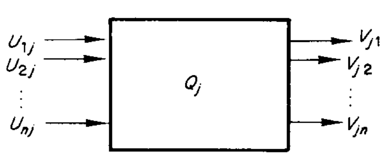

FIG. 1. General production relations. U_{ij} is the input of commodity i to process j. V_{ji} is the output of commodity i from process j.

In the Leontief system there would be only one V_{ji} for each process (j) and each commodity (i) would correspond to one process ($V_{ji}=0$ if $i \neq j$). We will assume a linear transformation process, or one where:

$$\begin{bmatrix} \lambda U_{1j}, \lambda U_{2j}, \ldots, \lambda U_{nj} \\ \lambda V_{j1}, \lambda V_{j2}, \ldots, \lambda V_{jn} \end{bmatrix}$$

or $(\lambda U_{ij}, \lambda V_{ji})$ holds for all λ.† Next we set up a system of m linear equations (one for each commodity) which balance the total input of each commodity to all processes against the total output of each commodity from all processes. Note that m does not necessarily equal n, that is, there can be a different number of commodities and processes. Thus we have:

$$\begin{array}{l} U_{11} + U_{12} + \ldots + U_{1n} + Y_1 = V_{11} + V_{21} + \ldots + V_{n1} \\ U_{21} + U_{22} + \ldots + U_{2n} + Y_2 = V_{12} + V_{22} + \ldots + V_{n2} \\ \vdots \\ U_{m1} + U_{m2} + \ldots + U_{mn} + Y_m = V_{1m} + V_{2m} + \ldots + V_{mn} \end{array} \tag{1}$$

An "open" system allows for imports (commodities that are inputs from outside the system boundaries) and exports (commodities that are outputs to outside the system boundaries). We develop an open system by denoting imports and exports of commodity i by $U_{i,n+1}$ and $V_{n+1,i}$, respectively, and net exports by $Y_i = V_{n+1,i} - U_{i,n+1}$. Any change in the storage of a commodity (net accumulation or depletion) is included as a net export. Also, depreciation of an already produced commodity (as distinct from losses in the process of production) is considered to be a net export, since it represents a loss of the commodity to the system. This is an important distinction, since it implies that at steady state the net input to the system is just enough to maintain all the internal stocks of commodities against depreciation.

The left side of each equation in (1) represents the demand for each commodity (as the sum of input requirements for all processes plus exports or "final demand") while the right side represents the supply of each commodity (from internal production plus imports). Note, for comparison, that in the Leontief model the right-hand side of each equation is set equal to the total output of each commodity, and supply is implicitly assumed to meet or exceed demand.

Next we define two sets of technical coefficients: $A_{ij} = U_{ij}/X_j$ = gross input coefficient, the amount of commodity i required as input for process

† In practice we require that λ be small for the static equilibrium assumption. For dynamic applications we would require a more complex functional relation (probably nonlinear) relating inputs to outputs. As long as we limit applications to static accounting applications the linearity assumption is acceptable, however.

j per unit activity of process j. These are the Leontief technical coefficients. $B_{ij} = V_{ji}/X_j$ = gross output coefficient, the amount of commodity i produced as output from process j per unit activity of process j. X_j = some measure of the activity level of process j.† Thus we can rewrite equation (1) as:

$$\begin{array}{l} A_{11}X_1 + A_{12}X_2 + \ldots + A_{1n}X_n + Y_1 = B_{11}X_1 + B_{12}X_2 + \ldots + B_{1n}X_n \\ A_{21}X_1 + A_{22}X_2 + \ldots + A_{2n}X_n + Y_2 = B_{21}X_1 + B_{22}X_2 + \ldots + B_{2n}X_n \\ \vdots \\ A_{m1}X_1 + A_{m2}X_2 + \ldots + A_{mn}X_n + Y_m = B_{m1}X_1 + B_{m2}X_2 + \ldots + B_{mn}X_n \end{array} \tag{2}$$

In matrix notation we can indicate equations (2) as

$$\mathbf{AX} + \mathbf{Y} = \mathbf{BX} \tag{3}$$

or

$$(\mathbf{B} - \mathbf{A})\mathbf{X} = \mathbf{Y} \tag{4}$$

where $\mathbf{A}$ is a $n \times m$ matrix of gross input coefficients, $\mathbf{B}$ is a $n \times m$ matrix of gross output coefficients. If there are no joint products and $m = n$, $\mathbf{B}$ is the identity matrix. $\mathbf{X}$ is a m dimensional column vector of process activity levels. $\mathbf{Y}$ is a n dimensional column vector of "net exports" (including "exports" or losses due to depreciation and any net accumulation over the time interval).

Since we do not necessarily have equal numbers of commodities and processes or a one-to-one correspondence between commodities and processes, the above system of equations is not necessarily solvable using standard linear algebra manipulations and is best handled as a linear programming problem. Certain assumptions can be imposed to allow I–O models to handle joint products but these either (1) can lead to negative relative weights or (2) require that we know the relative weights *a priori*, both of which are unacceptable for our purposes. The former (commodity-technology assumption) assumes that joint products require the same inputs as when they are produced as the sole output from a process. Thus, one simply transfers the inputs required for the sole output process away from the joint output process. This can lead to "negative inputs" and negative relative weights. The second assumption (the market shares assumption)

† The appropriate activity measure in input–output analysis is the total output of the one commodity produced by each sector. In the LP model the appropriate activity measure is not obvious and there are several possibilities. We could choose a "main" product output, multiply all the outputs by weighting factors and add, etc. In our examples we use the first alternative but others are certainly possible. We have not investigated the effects of using different activity measures on the results.

assigns inputs to joint products based on the percentage of the total value of the output that each joint product represents (Ritz, 1979). To arrive at this percentage (market share), we must already know the relative weights to apply to the joint outputs. For economic applications market prices are used to determine market shares, but in ecological examples it is just these relative weights that we are trying to calculate and we cannot use them *a priori* to obtain a solution.

Linear programming allows a more general and flexible formulation of the problem. LP uses a system of linear inequalities (rather than equations) that represent constraints on the system's behavior combined with an objective function, that represents the direction of system evolution within the constraints. Systems of linear inequalities have many possible (feasible) solutions but only the solution that maximizes (or minimizes) the objective function is optimal. The choice of objective function is therefore critical to the definition of the problem and its solution. LP formulations have another desirable characteristic for this application. They consist of both a "primal" problem and a sort of mirror image problem implicit in the primal called the "dual". The dual problem is of considerable importance since it can be used to determine the "shadow prices" or implied valuations for ecological commodities. We can formulate the generalized von Neumann model as a classic primal/dual LP pair (Dorfmann *et al.*, 1958), with objective functions consistent with (but not identical to) Hannon's (1979) hypothesis† and also with Oster & Wilson's (1978) hypothesis of "maximum egronomic efficiency" and Odum's (1971) "maximum power" hypothesis.

Primal

Minimize

$$P = \mathbf{EX} \tag{5}$$

(primary resource cost of all processes) subject to:

$$(\mathbf{B} - \mathbf{A})\mathbf{X} \geq \mathbf{Y} \tag{6}$$

(materials balance constraints, one for each commodity) and

$$\mathbf{X} \geq 0 \tag{7}$$

(non-negativity constraints on the process activity levels).

† Hannon (1979, p. 271) states: "In brief, the hypotheses are that while the components of an ecosystem strive to maximize their total direct and indirect energy storage within the constraints of their production characteristics, the overall system strives to minimize the metabolized energy per unit of stored biomass energy."

Dual

Maximize

$$P' = \mathbf{eY} \tag{8}$$

("value" of net output) subject to

$$\mathbf{e}(\mathbf{B}-\mathbf{A}) \leq \mathbf{E} \tag{9}$$

(value balance constraints, one for each process) and $\mathbf{e} \geq 0$ (non-negativity constraints), where P is the value of the primal objective function, P' is the value of the dual objective function, $\mathbf{B}$ is an $n \times m$ matrix of commodity outputs per unit activity level and $\mathbf{A}$ is an $n \times m$ matrix of commodity inputs per unit activity level. $\mathbf{Y}$ is an $n \times 1$ vector of commodity net exports, $\mathbf{X}$ is a $m \times 1$ vector of process activity levels, $\mathbf{E}$ is a $1 \times m$ vector of net primary resource inputs by process, per unit activity level, and $\mathbf{e}$ is a $1 \times n$ vector of energy intensities (relative weights or shadow prices) per unit of commodity.

For ecological systems the primary (exogenous) resources ($\mathbf{E}$ vector) are direct solar energy and the solar energy embodied in other primary inputs (i.e., commodities which are not also produced inside the system). The $\mathbf{e}$ vector can thus be interpreted as the direct and indirect energy cost (embodied energy) per unit commodity with allowances for joint products and alternative production technologies. The system adjusts the process activity levels ($\mathbf{X}$) and the relative weights or embodied energy intensities ($\mathbf{e}$) in order to minimize the total primary resource (energy) cost of production, which also maximizes the total value (relative weights times quantities) of the net exports from the system.

4. Results

To demonstrate the LP model, we employ the same example system as Hannon (1973, 1976, 1979) and Herendeen (1981). Data on biomass and nitrogen flows for the Silver Springs, Florida, ecosystem were obtained originally by Odum (1957).

Table 1 shows the flow data for Silver Springs. Inputs (the upper entry) and outputs (the lower entry in parentheses) of commodities (listed along the left of the table) to and from processes (listed along the top of the table) are shown. Export of biomass was divided proportionally among all biomass commodities and not allocated solely to producers, as in Herendeen (1981). The flow network is shown diagrammatically in Fig. 2. Each commodity in the tables and figure is kept separate and we seek the conversion of weighting factors that would allow us to compare them.

TABLE 1

Input–output table for Silver Springs, Florida.

Commodity (units)	Processes: Primary producers	Herbivores	Carnivores	Top carnivores	Decomposers	Export (import)	Total demand (supply)
Plant biomass (g)	0	748·4	0	0	874·2	460·3	2082·9
	(1962·9)	(0)	(0)	(0)	(0)	(120)	(2082·9)
Herbivore biomass (g)	0	0	85·1	0	170·7	72·6	328·4
	(0)	(328·4)	(0)	(0)	(0)	(0)	(328·4)
Consumer biomass (g)	0	0	0	4·7	6·9	3·3	14·9
	(0)	(0)	(14·9)	(0)	(0)	(0)	(14·9)
Top carnivore +	0	0	0	0	80·6	22·9	103·5
decomposer biomass (g)	(0)	(0)	(0)	(1·3)	(102·2)	(0)	(103·5)
Nitrogen (g)	121·7†	0	0	0	0	4415·6§	4537·3
	(0)	(0)	(0)	(0)	(70·3)‖	(4467·0)‡	(4537·3)
Heat (kcal)	0	0	0	0	0	407 986	407 986
	(401 167)	(1890)	(316)	(13)	(4600)	0	(407 986)
Sun (kcal)	410 000	0	0	0	0	0	410 000
	(0)	(0)	(0)	(0)	(0)	(410 000)	(410 000)

Upper numbers are inputs and lower numbers (in parentheses) are outputs of commodities listed along the left to processes listed along the top. All flows are $/m^2yr$.

† Nitrogen comprises 6·2% of plant biomass by weight. Net primary production is 2082·9 g/m^2yr.

§ Net output of N obtained by difference (total input-uptake).

‖ Nitrogen produced by decomposers is 6·2 percent of biomass input to that sector. Assume all N in decomposing biomass is regenerated.

‡ $3{\cdot}395 \times 10^8$ g/yr inflow in spring water distributed over an area of $7{\cdot}6 \times 10^4$ m^2.

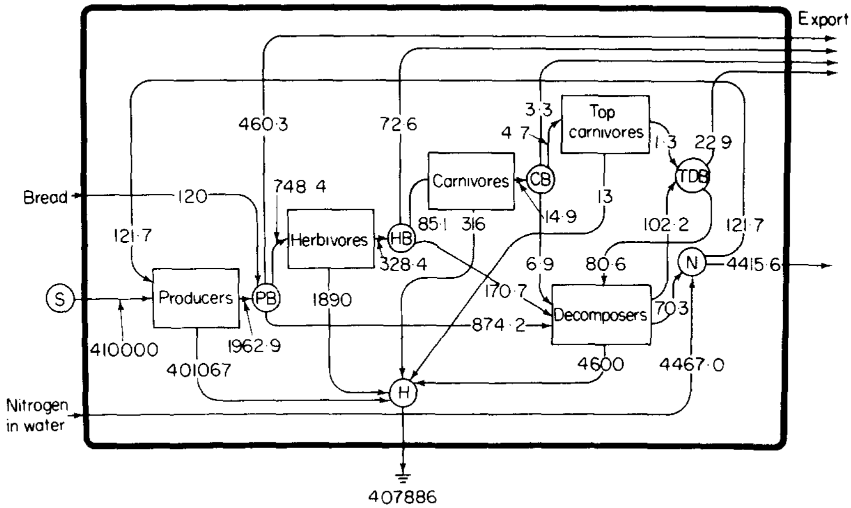

FIG. 2. Transactions diagram for the Silver Springs ecosystem. Boxes represent production processes. Circles represent commodities. PB = plant biomass: HB = herbivore biomass; CB = carnivore biomass; TDB = top carnivore + decomposer biomass; N = nitrogen; all g/m^{2}yr. H = waste heat; S = sunlight (both kcal/m^{2}yr).

The table and figure correspond to the mathematical description outlined earlier. Table 1 lists the demand or use of each ecological commodity (U_{ij}) as the sum of inputs to internal processes and export. For example, of the total 2082·9 g/m^{2}yr plant biomass available for use, 748·4 g/m^{2}yr was used as input to herbivores (U_{12}), 874·2 g/m^{2}yr was consumed by decomposers (U_{15}) and 460·3 g/m^{2}yr was exported (U_{16}). Table 1 also shows the supply characteristics of each commodity in the system as the sum of internal production and imports. For example, of the total 4537·3 gN/m^{2}yr produced or imported into the system, 70·3 gN/m^{2}yr was produced internally by decomposers (V_{55}) and 4467·0 gN/m^{2}yr was imported into the system in the inflowing water (V_{65}).

In Fig. 2 each process is represented by a box and each commodity by a circle. Looking at the inputs and outputs to a circle is equivalent to looking at the corresponding commodity rows in Table 1. Looking at a box in the figure is equivalent to looking at the corresponding process column in Table 1. The system receives a solar energy input of 410 000 kcal m^{2}yr. The input of 120 g/m^{2}yr of bread thrown into the system by tourists was considered a net import of plant biomass.

The essence of the valuation problem is finding a set of weighting or conversion factors that would allow us to add and balance process inputs and outputs (boxes in Fig. 2 or columns in Table 1) while maintaining our ability to add and balance commodities (circles in Fig. 2 or rows in Table 1).

Table 1 and Fig. 2 show waste heat as a secondary joint product of all processes. Decomposers produce their own biomass, nutrients, and waste heat as joint products. Other joint products, such as feeding or nesting sites, may be included in the model as data permit.

The treatment of waste heat is critical since it determines whether a first law, a reversible second law, or an irreversible second law analysis results.

TABLE 2

Energy intensities (in kcal/g) for the Silver Springs ecosystem for various values of the quality factor for waste heat

Quality factor for waste heat†	Plant biomass	Herbivore biomass	Consumer biomass	Top carnivore + decomposer biomass	Nitrogen
0·0	297·05	676·95	3866·32	13 978·24	1422·20
0·01	294·16	670·44	3829·35	13 844·66	1408·60
0·0428	284·68	694·02	3707·73	13 405·31	1363·91
0·1	287·17	611·71	3495·85	12 639·84	1286·07
0·5	152·67	350·80	2014·19	7 287·07	741·78
1·0	4·50	4·50	4·50	4·53	0·0

† This is operationally equivalent to Herendeen's (1981) α.

Table 2 shows the energy intensities which are analogous to shadow prices, that result from application of the LP model.† The quality factor or "value" of waste heat was varied over the range from zero to one. (Different quality factors for heat were handled by converting the heat to solar equivalents (by multiplying by the chosen quality factor) and decreasing the input of solar energy by that amount.) A quality factor of one for waste heat implies a first law accounting since no distinction is made between dissipation and production. A quality factor of zero for waste heat implies a reversible second law accounting since the heat outflow is assumed to be completely dissipated and thus valueless. An intermediate quality factor implies an irreversible second law accounting since waste heat is assumed to be incompletely dissipated and to still contain some available energy and some "value".

† The ZX3LP subroutine of the IMSL library was used to perform the analysis on the IBM 370/3033 at LSU. The subroutine uses a revised simplex technique.

5. Discussion

This paper demonstrates the application of linear programming techniques to the valuation problem in ecological systems. Joint products, which are unavoidable in any system that contains sorting or dissipation activity, can be dealt with in the LP format, but simply casting the data in this formal structure does not guarantee intelligible results. The distinction between dissipation (change in structure in the direction of increasing entropy) and production (sorting and assembly, or change in structure in the direction of decreasing entropy) is critical. If dissipation products (such as waste heat) are not distinguished from production products, then a first law accounting results. If they are distinguished, then a more useful second law accounting results. Hannon (1973, 1976, 1979), following established conventions in ecology, used a quality factor of one for waste heat and therefore obtained constant energy intensities. Herendeen (1981) attempted to draw the distinction between dissipation and production by eliminating respiratory heat flow from the production matrix (see also Ulanowicz, 1972). Herendeen's results show increasing energy intensities moving up the food chain as we might expect. Our results extend Herendeen's analysis by allowing for non-dissipative joint products. As a first step we include nitrogen in the system. Contrary to Herendeen's expectation the inclusion of nitrogen did not tend to decrease the variance in energy intensities but rather increased it.

Herendeen's α factor is operationally equivalent to the quality factor for waste heat. A value of $\alpha = 1$ implies a first law accounting and energy intensities equal to the calorimetric values for the commodities. A value of $\alpha = 0$ implies a reversible second law accounting. An intermediate value for α may be more appropriate for real irreversible systems since it implies that the dissipation products are not completely degraded or that all the dissipated potential is not embodied in the products.

When waste heat is the main dissipation product the Carnot ratio using the source-sink temperature gradient may be an appropriate independent measure of α. For example, we might define an index of α as:

$$\alpha' = 1 - ((T_2 - T_1)/T_2) = T_1/T_2 \tag{9}$$

where T_1 = temperature of the source and T_2 = temperature of the sink.

The implication of this formulation is that waste heat (at sink temperature T_1) still has work potential relative to the universe, which is at a still lower temperature (≈ 3 K). For our Silver Springs example, we calculated a value for α' based on $T_2 = 7000$ K (effective temperature of sunlight) and $T_1 = 300$ K (average temperature of the earth) as $\alpha' = 1 - ((7000\text{–}300)/7000) = 0{\cdot}0428$.

The energy intensities based on this α value are included in Table 2. These show an average variation from the intensities calculated with $\alpha = 0$ of 0·96%. Thus, assuming that $\alpha = 0$ does not lead to significant numerical error in this example since more than 95% of the available energy in sunlight is degraded by the time it emerges as 300 K waste heat.

The problem with this formulation is that Carnot ratios are based on reversible (infinitely slow) reactions. The question of what percentage of the dissipated gradient is embodied in the products is still not adequately addressed in this formulation (100% embodiment is assumed implicitly). Odum & Pinkerton (1955) have suggested and defended with theoretical arguments an embodiment ratio closer to 50%. In our example, an $\alpha = 0{\cdot}5$ would lead to energy intensities with an average variation of 48·1% compared to $\alpha = 0$.

Inspection of Table 2 shows that, except for values close to zero (second law case), changes in the value of α change the absolute values of the energy intensities more than their relative values. Thus, questions about relative energy intensities are not as dependent on the choice of α as are questions of absolute energy intensities. This may prove useful in resource management applications, since we could "scale" the model using known resource prices without changing the relative energy intensities significantly. It also means that the choice of α does not significantly affect questions about relative economic value (Costanza, 1980) since changing α affects mainly the absolute, not the relative, energy intensities.

6. Zero Energy Intensities and Activity Levels

While the non-negativity constraints in the LP model prevent negative energy intensities, an unequal number of commodities and processes invariably leads to either zero activity levels for some processes or zero energy intensities for some commodities. These results are interpretable within the framework of linear programming as indicating an uncompetitive process or the existence of "slack" or unused capacity in the system (cf. Dorfman, Samuelson & Solow, 1958). We avoided this result by aggregating top carnivore and decomposer biomass (since they are both at the "top" of the food chain). This allowed an equal number of commodities (four biomass types and nitrogen) and processes (five tropic levels) and produced positive nonzero activity levels and energy intensities for all components. An equal number of commodities and processes does not guarantee this result, however. The "structure" of the data must also be such that all processes and commodities are part of what the LP model deems to be the optimal system. In our Silver Springs example, it was possible to change individual entries in the I–O table slightly and cause processes and commodities to be eliminated from the optimal solution. Since Silver Springs is a stable ecosystem that has persisted in essentially the same condition for many years, one would expect its structure to be very close to optimal. An optimization model that does not reproduce this conclusion brings into question the model, the data, or both.

One interpretation is that the model is correct and any failure to reproduce the real system is caused by imprecise data. Since the ecological data used to construct I–O tables are admittedly very imprecise, this assumption seems reasonable. Given this assumption, the LP model could be used as a consistency check on the data. One way to handle this problem would be to specify a range of certainty for each individual entry in the I–O table and use a Monte Carlo approach to determine which combinations of numbers randomly selected from within these ranges lead to all non-zero energy intensities and activity levels. An exercise like this could be used to construct ranges on the resulting energy intensities and activity levels.

Since the I–O model does not constrain energy intensities to be nonnegative, the same data that produce zero energy intensities (with $m = n$) in the LP model will produce negative energy intensities in an I–O formulation. If the I–O formulation (with $m = n$) produces all positive energy intensities, its results will be identical to an LP formulation using the same data. As noted previously, the I–O and LP models are very similar, but the LP model makes the optimization aspects of the problem more visible.

7. Resource Management Applications

Input–output type models that summarize ecological transformations have been suggested as potentially useful to determine "shadow prices" for nonmarketed ecological commodities (Daly, 1968; Isard, 1972; Victor, 1972; Ayres, 1978; Costanza, 1980, 1982). This approach to shadow pricing has some potential advantages over the currently more popular "willingness-to-pay" formulations (Freeman, 1979). Most importantly, one does not have to interject humans into transactions in which they play no "direct" role and therefore exhibit irrelevant preferences. For example, people's willingness-to-pay for detritus is not meaningful since they do not use it directly. We must either survey detritivores about their preferences or carry the chain of production relations forward to the point where economically important commodities with well-defined markets are produced.

Linear programming models represent a well-developed, comprehensive way of detailing a complex set of production relations. As we have shown, they are applicable to ecological systems with joint products. The "shadow prices" that they produce are also interpretable as the direct and indirect primary resource cost of each product. Since sunlight is the major net input to the biosphere we may be able to use embodied solar energy as a common denominator linking ecological and economic systems (Costanza, 1980; Costanza & Neill, 1981). Such a common denominator would obviously be very useful for a broad range of resource management applications.

For realistic applications we would require detailed transactions tables for the ecological systems of the U.S. and the world. While these tables would always be inaccurate and incomplete (just as economic I–O tables always are), they would at least serve as a framework in which to summarize the extent of our current quantitative knowledge on ecological transactions, interdependence, and value. They would also be valuable in pinpointing areas for further research.

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