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ARTICULATION, ACCURACY AND EFFECTIVENESS OF MATHEMATICAL MODELS: A REVIEW OF FRESHWATER WETLAND APPLICATIONS

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ABSTRACT

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Eighty-seven mathematical models of freshwater wetlands and shallow water bodies were classified by wetland type, location, and degree of nonlinearity, and rated by three new indices: articulation; accuracy; and effectiveness. Articulation measures the size and complexity of the model in the three modes of components, space, and time. Accuracy combines measures of goodness-of-fit in each mode. Effectiveness measures explanatory power as a combination of articulation and accuracy. For the models reviewed accuracy was seen to fall with increasing articulation, probably as a result of increasing complexity and cost. Effectiveness, however, rose to a maximum at intermediate articulation and then fell, reflecting the fact that highly accurate models tended to be low in articulation (they said much about little), while highly articulate models tended to be low in accuracy (they said little about much). These methods for ranking models may prove useful for further analysis and the results of this analysis may provide a useful guide to model builders concerned with maximizing the effectiveness of their models using limited resources.

INTRODUCTION

Mathematical models are essential tools for understanding and managing ecosystems. There are a large number of models in the international literature on freshwater wetlands alone, and the list for general ecosystems models is enormous.

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One of the most fundamental questions facing scientists is: how does one evaluate alternative explanations (models), given that an essentially infinite number of models are possible and no one model can ever achieve perfection? In this paper we (a) develop several methods for classifying models and several scales for ranking models; (b) apply them to mathematical models of freshwater wetlands; and (c) interpret the results.

The task can be thought of as a systematic literature review. To access, evaluate, and use modeling information, periodic literature reviews are necessary. An effective review must be more than a list, however. It must organize and summarize the information in a convenient and useful way. In this review we summarize the models by:

(1) wetland type;
(2) geographical area;
(3) degree of nonlinearity;
(4) a measure of model complexity we call the 'degree of articulation' in terms of components, space, and time;
(5) where possible, indices of 'descriptive accuracy' and 'effectiveness'.

The review is limited to models that use some kind of formal mathematical description, either explicit equations or system diagrams with implied equations. The review is also limited to freshwater wetlands and shallow bodies of water. Shallow is taken to mean a maximum depth of 3 m or less. Rivers and other flowing water systems are also excluded.

While this review is not exhaustive, even of this limited subset of ecosystems, it covers 87 published works that span the modeling spectrum. Our organizational scheme provides a compact and accessible guide to this literature, and a method for classifying the models in terms that are useful for further analysis. First we briefly discuss some theoretical and philisophical points.

Mathematical tools

The mathematical tools available for ecological modeling can be classified in several ways, but the most frequently used classification is into linear and nonlinear categories. This distinction is important since linear systems theory is a well-developed branch of mathematics, while there is no corresponding body of theory on nonlinear systems (cf. Patten, 1976).

With the development of inexpensive high speed computing, nonlinear systems began to receive more attention. Computer algorithms are the tools associated with nonlinear systems instead of the more traditional theorems and proofs associated with linear theory. For example, the diagrammatic languages developed by Odum (1971) and Forrester (1961) are tools for developing computer programs to simulate nonlinear differential equation systems on computers. Even with readily available computers, however, nonlinear systems of equations are usually more difficult to manipulate than linear systems, and the more nonlinear a system of equations is, the more troublesome and difficult it usually is. This extra 'cost' must be compensated for by some 'benefits' in terms of a better description of the system. For this reason we looked at the 'degree of nonlinearity' of the models (measured as the percentage of nonlinear terms in the model equations) as one index of their mathematical difficulty.

Modes of articulation

To 'articulate' in language is to 'divide into syllables or words meaningfully arranged'. The term articulation is a useful concept for the discussion of ecosystem models since models are simplifications or divisions of the continuum that is nature. The simplification or 'dividing into words' can be accomplished in three modes: components; space; and time. Ecosystem models can be classified according to their degree of articulation in these.

Dynamic models are those articulated in time: spatial models are articulated in space; and compartment models are articulated in the system components (state variables). Of course, models may be articulated in all three modes at once. The more articulated a model is, the more expensive it is to build and run. Since no practical model can be maximially articulated in all three modes simultaneously, there are trade-offs among the modes. The optimal solution is determined by the relative importance of articulation in each mode to specific questions the model is designed to address.

Descriptive accuracy

The terms descriptive and predictive recur in the modeling literature. In general, descriptive refers to models that describe an existing structure or known behavior of a system. Predictive refers to models that are used to extrapolate the structure or behavior of a system outside the existing data boundaries. Most mathematical models combine both functions, since prediction is usually accomplished by mathematically manipulating the descriptive model. Another way of thinking of the relationship is that descriptive models are the mathematical tools available to model builders for interpolation, while prediction requires extrapolation beyond the existing data.

One reason for the abundance and diversity of both descriptive and predictive models of ecosystems is that no single model can claim a high degree of accuracy over a broad range of ecosystem structure or behavior. This is to be expected, because ecosystems are complex phenomena. The range of ecosystem structure and behavior cannot be totally captured in a single, cost-effective model. It is more useful to maintain a family of descriptive models (mathematical tools) and the predictions based on them (model uses), since each model has its own areas of accuracy and applicability.

The degree of accuracy with which a particular model can describe the historical structure or behavior of an ecosystem is measurable in a number of ways (Jørgensen 1982). Statistical indices of 'goodness-of-fit' (like R-squared) are useful in this regard. Frequently, however, the data necessary to calculate these statistical indices are not available, and other less formal methods must be used.

Although predictive accuracy is the goal of most models, descriptive accuracy is all one can measure before the fact. Descriptive accuracy is usually taken as at least a necessary (though not a sufficient) condition for predictive accuracy. As such, it is a useful concept for classifying model performance.

METHODS

We supplemented our existing knowledge of the wetland modeling literature with a systematic computer literature search and use of previous SCOPE wetland modeling reviews (i.e. Mitsch et al., 1982). While we cannot claim to have found via this method every reference that exists on freshwater wetland models, we do think that we achieved a relatively thorough review. Using this review, we compiled a bibliography and additional data for each reference. Wetland type (Table I), geographical location, and degree of nonlinearity were cataloged for each model (see Table II). In addition, indices of articulation, descriptive accuracy, and effectiveness, as described below, were computed for those models with sufficient information.

TABLE I

Wetland types used in this study

Number	Wetland type [a]
(1)	Forested swamp
(2)	Bottomland hardwood forest
(3)	Emergent marsh
(4)	Floating marsh
(5)	Shallow ponds and lakes (3 m or less)
(6)	Bogs and fens
(7)	Tundra
(8)	Combinations of the above

[a] After Mitsch et al. (1982).

Articulation indices

To compare the degree of articulation of the models, we constructed indices for the three major modes of articulation: components, space, and time. The index for each mode, we thought, should have a 'diminishing return' form, to reflect the fact that initial articulation effort is more productive than later effort, and that infinite effort asymptotically approaches a maximum articulation. We calculated the index for both the model and the data in all cases, since it is possible to construct a very articulate model with very inarticulate (or even nonexistent) data.

We used the following equation for the articulation index since it is a popular and simple form that exhibits the desired diminishing return behavior (although other equation forms that also exhibit diminishing returns would probably work just as well):

$$A_i = \frac{N_i - 1}{k_i + (N_i - 1)} \times 100 \tag{1}$$

where A_i = articulation index for mode i; N_i = number of divisions in mode i; k_i = scaling factor for mode i.

The number of divisions in each mode are: the number of components or state variables for the component mode (N_c), the number of time steps for the time mode (N_t), and the number of spatial units (i.e., pixels) for the space mode (N_a). The scaling factor was chosen to reflect the relative degree of difficulty of increasing the number of divisions in the mode, and an idea of the maximum size of the most articulated existing models in each mode. The scaling factors chosen for components, time, and space, respectively, were:

$k_c = 50$

$k_t = 1000$

$k_s = 5000$

These parameter values indicate that adding a component (state variable) to a model is much more difficult than adding a time step, which is in turn more difficult than adding a pixel of spatial resolution. For example, a model with 50 state variables, 1000 time steps, and 5000 spatial units would have an articulation index of 50 for each mode, and 50 average. The same average articulation could be achieved by a model with 200 state variables, 2340 time steps, and 1 spatial unit.

The distinction between the articulation index of the model and data is critical. It is relatively easy to run a simulation model with 10 000 time steps (or infinite time steps on an analog computer) but it is very difficult to collect supportive data at this frequency. In many cases, the articulation of the data is the limiting factor.

Descriptive accuracy

An index of descriptive accuracy was calculated as the percentage of the total (historical) variation that was explainable by the model, averaged over all three modes and stated as a fraction between 0 and 1. The average value was used to standardize the index across all three modes of articulation and to estimate model accuracy as a percentage of the total maximum accuracy possible.

Our ability to calculate this index varies from study to study. In many cases the necessary information was not available. In most of the remaining cases, only a rough estimate of the descriptive accuracy of the model was possible. In only a few cases there was either enough information to precisely calculate the index or it has already been calculated and reported. We noted which of these three situations applied for each model.

Two situations arose frequently: (a) curves showing the model's behavior in time would be given, with the associated data points; or (b) the steady-state values for the model components would be given with average values for the real system. In the first case we estimated the descriptive accuracy of the model by using a small digitizer connected to a microcomputer to replicate the model and data time series. We then calculated the coefficient of determination (R^2) for each component and took the average; the error associated with this procedure was about 5–10% (based on the precision of the digitizer). In the second case we simply calculated the percent deviation between the model and the data as:

$$1 - \frac{|\text{model} - \text{data}|}{\text{data}} \tag{2}$$

and took the average over the components or spatial units.

Neither of these procedures gives very precise estimates, and this must be remembered when interpreting the results.

Effectiveness

The best model is the one that explains the most. A model with a high descriptive accuracy does not explain much if its accuracy is limited in articulation, however. For example, a one component model may have high descriptive accuracy for that one component, but say nothing about the rest of the system. Similarly, a highly articulated model which does not fit the data explains little.

To rank the models we developed an index of effectiveness or explanatory power. This index was calculated as the coefficient of determination (if calculatable) for each mode multiplied by the minimum of the data or model articulation index for that mode, and then averaged over the three modes. The most effective model under this scheme is one that balances the costs of added articulation with the benefits of increased accuracy to do the best job of explaining all the modes of the system.

RESULTS

Our review turned up 87 models in 59 different studies that were (a) sufficiently documented and (b) within the limits of our review. There were several additional models that were tangential to the topic, too general to allow calculation of indices, redundant with models we had already reviewed, or lacked readily available documentation. The reviewed models are a representative, though not an exhaustive, sample.

Tables II–IV summarize the results of our review. Table II lists the reviewed models, sorted according to wetland type (as given in Table I), and lists the location, principal modeling unit, and the raw scores used for calculating the articulation indices and other parameters for each model. Many of the reviewed models had missing values for some of the data. These are indicated by a dash (–) in Table II and in subsequent tables.

Table II indicates that the majority of the modeling effort has been focused on shallow lakes (30 models), forested swamps (18 models), and emergent marshes (14 models). Floating marshes (5 models), tundra (4 models) and bogs and fens (2 models) are noticeably underrepresented. Except for some of the shallow lake models, the modeled systems are all in the United States.

The principal modeling component (PMC) is listed for those models that dealt with mainly one accounting unit that cycled through the internal modeling components. For example, there were 13 hydrologic models (PMC = water), 13 biomass models, 4 carbon models, 4 energy models, and 10 phosphorus models. There were several models that dealt in multiple units and these were labled PMC = general.

Table II lists the number of components or state variables in the model (MCA) and the data (DCA), the number of spatial units or pixels in the model (MSA) and the data (DSA), and the number of time intervals in the model (MTA) and the data (DTA) for each study. 'Analog' models or data are given a value of 99999. Different types of models have characteristic values for these variables, but these are more easily seen after the articulation indices based on them are calculated.

TABLE II

Reviewed models sorted by wetland type (see Table I), showing the date and location of the study and raw scores used to calculate indices

Author	Year	Region	State	Nation	PMC	MCA	DCA	MSA	DSA	MTA	DTA	PCNL	$\overline{FIT}$	FIT-CALC	CFM	SFM	TFM
(1) *Forested swamp*																	
Brown	1978.1	Green Swamp	FL	U.S.A.	Water	4	2	1	1	22500	108	41.6	0.94	1	1	0	1
Brown	1978.2	Cypress dome	FL	U.S.A.	Biomass	10	5	1	1	1	24	15.8	0.00	0	0	0	0
Brown	1978.3	Floodplain	FL	U.S.A.	Biomass	9	4	1	1	1	24	15.8	0.00	0	0	0	0
Costanza	1975.1	Green Swamp	FL	U.S.A.	General	7	7	1	99999	1	1	26.0	0.00	0	0	0	0
Costanza et al.	1983.1	Deltaic Plain	LA	U.S.A.	General	110	110	1	273	1	1	0.0	0.00	0	0	0	0
Ewel and Deghi	1978.0	Gainesville	FL	U.S.A.	Phosphorus	15	15	1	2	10000	1	19.0	0.00	0	0	0	0
Florschutz	1978.0	Southwest	FL	U.S.A.	Phosphorus	4	4	1	1	99999	1	10.0	0.00	0	0	0	0
Littlejohn	1977.0	Naples Bay	FL	U.S.A.	Water	2	2	1	12	99999	–	0.0	0.00	0	0	0	0
Mitsch	1983.0	Cypress Dome	FL	U.S.A.	Biomass	8	7	1	4	9999	6	41.7	–	–	–	–	–
Nessel	1978.0	Waldo	FL	U.S.A.	Phosphorus	14	1	1	1	1	0	40.0	0.00	0	0	0	0
Ogawa	1977.0	Southern	IL	U.S.A.	General	15	15	1	1	1	1	50.0	0.00	0	0	0	0
Paschal et al.	1979.0	Dismal Swamp	NC	U.S.A.	Mice	11	11	1	100	26	4	50.0	0.16	1	1	0	1
Patten and Matis	1982.0	Okefenokee	GA	U.S.A.	Water	4	4	1	1	1	1	0.0	0.00	0	0	0	0
Rykiel	1977b.1	Okefenokee	GA	U.S.A.	Water	2	2	1	1	1	1	0.0	0.00	0	0	0	0
Rykiel	1977a.2	Okefenokee	GA	U.S.A.	Water	8	8	1	1	1	1	0.0	0.00	0	0	0	0
Sklar	1983.0	Barataria	LA	U.S.A.	Water	1	1	1	4	768	24	45.2	0.50	1	1	0	1
Wharton et al.	1976.0	General	G	U.S.A.	General	17	0	1	0	99999	0	75.0	0.00	0	0	0	0
Wiemhoff	1977.0	Southern	IL	U.S.A.	Water	2	2	1	1	104	1	0.0	0.00	0	0	0	0
(2) *Bottomland hardwood forest*																	
Botkin et al.	1972.0	Hubbard Brook	NH	U.S.A.	Trees	13	13	99999	99999	200	1	20	0.3	1	1	0	0
Costanza et al.	1983.2	Deltaic Plain	LA	U.S.A.	General	16	1	1	273	1	1	0	0.0	0	0	0	0
Kuenzler et al.	1980.1	Creeping Swamp	NC	U.S.A.	Carbon	7	7	1	1	1	1	0	–	–	–	–	–
Kuenzler et al.	1980.2	Creeping Swamp	NC	U.S.A.	Phosphorus	9	9	1	1	1	1	0	–	–	–	–	–
Odum and Brown	1975.0	Green Swamp	FL	U.S.A.	General	35	1	1	99999	1	1	80	0.0	0	0	0	0
Phipps	1979.1	White River	AR	U.S.A.	Trees	25	25	4356	4356	200	1	43	0.0	0	0	0	0
Phipps	1979.2	White River	AR	U.S.A.	Trees	25	25	4356	4356	200	1	43	0.0	0	0	0	0
Phipps	1979.3	White River	AR	U.S.A.	Trees	25	25	4356	4356	200	1	43	0.0	0	0	0	0
Phipps	1979.4	White River	AR	U.S.A.	Trees	25	25	4356	4356	200	1	43	0.0	0	0	0	0
(3) *Emergent marsh*																	
Bayley and Odum	1976.0	Everglades	FL	U.S.A.	General	4	2	1	1	99999	1	25	0.00	0	0	0	0
Burns and Taylor	1979.0	Kissimmee R.	FL	U.S.A.	Water	1	1	135	1	99999	1	0	0.00	0	0	0	0
Burns and Taylor	1979.1	Kissimmee R.	FL	U.S.A.	Phosphorus	7	7	1	1	99999	1	8	0.88	1	1	0	0
Cleveland et al.	1981.0	Southern	LA	U.S.A.	Area	4	4	1	273	100	3	43	0.00	0	0	0	0
Costanza	1975.0	Green Swamp	FL	U.S.A.	General	7	7	1	99999	1	1	26	0.00	0	0	0	0

Costanza et al.	1983.3	Deltaic Plain	LA	U.S.A.	General	70	70	1	273	1	1	0	0.00	0	0	0	0
Costanza et al.	1983.6	Deltaic Plain	LA	U.S.A.	General	15	1	1	263	1	1	0	0.00	0	0	0	0
Flebbe	1982.1	Okefenokee	GA	U.S.A.	Carbon	5	5	1	1	1	1	0	0.00	0	0	0	0
Flebbe	1982.2	Okefenokee	GA	U.S.A.	Carbon	5	5	1	1	1	1	0	0.00	0	0	0	0
Flebbe	1982.3	Okefenokee	GA	U.S.A.	Carbon	5	5	1	1	1	1	0	0.00	0	0	0	0
Gardner et al.	1980.0	Lake Wingra	WI	U.S.A.	Water	2	2	1	1	244	244	50	0.89	1	1	0	1
Huff and Young	1980.0	Madison	WI	U.S.A.	Water	1	1	1	1	240	240	0	0.86	1	0	0	1
Stone and McHugh	1979.0	Barataria	LA	U.S.A.	Water	3	3	3025	5	99999	120	0	0.69	1	1	0	0
White et al.	1978.0	Navarre	OH	U.S.A.	Tritium	1	1	31	6	2600	28	0	0.58	1	0	1	1
(4) *Floating marsh*																	
Mitsch	1976.0	Lake Alice	FL	U.S.A.	Water	1	1	1	1	269	269	0.0	–	–	–	–	–
Mitsch	1976.1	Lake Alice	FL	U.S.A.	General	21	15	1	3	99999	7	47.8	0.7	1	1	0	1
Sklar	1983.1	Barataria	LA	U.S.A.	Biomass	7	6	1	1	7200	24	45.2	0.4	1	1	0	1
Sklar	1983.2	Barataria	LA	U.S.A.	Biomass	7	6	1	1	1	24	45.2	0.0	0	0	0	0
Sklar	1983.3	Barataria	LA	U.S.A.	Biomass	7	6	1	1	1	24	45.2	0.0	0	0	0	0
(5) *Shallow lakes*																	
Browder	1978.0	Southwest	FL	U.S.A.	General	10	15	1	60	300	1	13.0	–	–	–	–	–
Costanza et al.	1983.4	Deltaic Plain	LA	U.S.A.	General	38	38	1	273	1	1	0.0	0.00	0	0	0	0
Costanza et al.	1983.5	Deltaic Plain	LA	U.S.A.	General	14	1	1	273	1	1	0.0	0.00	0	0	0	0
Halfon	1979.0	Heart Lake	ONT	Canada	Phosphorus	4	2	1	1	300	27	0.0	0.95	1	1	0	1
Huff et al.	1973.0	Lake Wingra	WI	U.S.A.	Biomass	8	8	1	1	1750	24	50.0	0.74	1	0	0	1
Jolánkai	1982.1	Lake Balaton		Hungary	TP	1	1	1	20	1	60	0.0	0.85	1	0	0	0
Jolánkai	1982.2	Lake Balaton		Hungary	TN	1	1	1	20	1	60	0.0	0.85	1	0	0	0
Jørgensen	1982.0	Copenhagen		Denmark	Eutrophic	17	17	1	0	3650	35	35.8	0.69	2	1	0	1
Loucks and Weiler	1979.0	Lake Wingra	WI	U.S.A.	Phosphorus	18	16	1	–	–	–	50.0	–	–	–	–	–
Loucks and Weiler	1979.0	Lake Wingra	WI	U.S.A.	Phosphorus	18	16	1	–	–	–	50.0	–	–	–	–	–
Mitsch	1975.0	Cypress Pono	FL	U.S.A.	Biomass	5	5	1	4	99999	6	40.0	–	–	–	–	–
Nyholm	1978.1	Lake Ollemp		Denmark	General	7	7	1	1	360	9	50.0	0.23	1	1	0	1
Ondok and Pokorny	1982.0	S. Bohemia		Czechoslovakia	Oxygen	4	3	1	1	384	80	14.3	0.88	2	1	0	1
Richey	1977.0	Mountains	CA	U.S.A.	Phosphorus	9	7	5	6	6	6	33.3	0.50	1	1	1	0
Svirezhev and Voinov	1982.0	General		U.S.S.R.	Eutrophic	4	0	1	0	99999	0	22.2	0.00	0	0	0	0
Uhlmann and Recknagel	1982.0	General		G.D.R.	Bod	1	1	1	12	1	40	0.0	0.90	1	1	0	0
Verhagen	1978.0	De Grote Rug		The Netherlands	Biomass	8	8	1	1	9999	6	0.0	–	–	–	–	–
Walters	1980.0	General			Biomass	11	0	1	0	540	0	25.0	0.00	0	0	0	0
Walters et al.	1980.1	Lake George		Uganda	General	28	28	1	1	36500	365	50.0	0.10	1	1	0	1
Walters et al.	1980.2	Lake Wingra	WI	U.S.A.	General	28	28	1	1	36500	365	50.0	0.38	1	1	0	1
Wiegert	1971.1	Yellowstone	MT	U.S.A.	Algae-fly	6	6	1	3	360	1	50.0	0.85	0	0	0	0
Wiegert	1971.2	Yellowstone	MT	U.S.A.	Algae-fly	6	6	1	3	360	1	50.0	0.68	2	1	0	0
Wiegert	1971.3	Yellowstone	MT	U.S.A.	Algae-fly	6	6	1	3	360	1	50.0	0.78	2	1	0	0
Wiegert	1971.4	Yellowstone	MT	U.S.A.	Algae-fly	4	6	1	3	360	1	50.0	0.19	2	1	0	0
Wiegert	1971.5	Yellowstone	MT	U.S.A.	Algae-fly	2	6	1	3	360	1	50.0	0.69	2	1	0	0

TABLE II (continued)

Author	Year	Region	State	Nation	PMC	MCA	DCA	MSA	DSA	MTA	DTA	PCNL	FIT	FIT-CALC	CFM	SFM	TFM
Wheeler et al.	1978.0	Central	IL	U.S.A.	Lead	6	6	1	1	48	1	0.0	0.82	2	1	0	0
Williams	1971.1	Cedar Bog Lake	MN	U.S.A.	Energy	3	11	1	1	99999	1	0.0	0.00	0	0	0	0
Williams	1971.2	Cedar Bog Lake	MN	U.S.A.	Energy	10	10	1	1	99999	1	0.0	0.00	0	0	0	0
Williams	1971.3	Cedar Bog Lake	MN	U.S.A.	Energy	10	10	1	1	99999	1	29.0	0.00	0	0	0	0
Williams	1971.4	Cedar Bog Lake	MN	U.S.A.	Energy	10	10	1	1	99999	1	29.0	0.00	0	0	0	0
(6) *Bogs and fens*																	
Bazilevich and Tishkov	1982.0	Novgorod		U.S.S.R.	General	28	28	1	1	1	1	50	–	–	–	–	–
Silvola and Hanski	1979.0			Finland	Peat	1	0	1	0	72	0	0	0	0	0	0	0
(7) *Tundra*																	
Dauffenbach et al.	1981.0	North Slope	AK	U.S.A.	General	51	0	1	0	–	0	50.0	0.00	0	0	0	0
Douce	1978.0	Barrow	AK	U.S.A.	Biomass	5	5	1	–	99999	–	6.9	0.00	0	0	0	0
Miller et al.	1976.0	Barrow	AK	U.S.A.	Phosphoru	9	4	1	1	292	100	50.0	0.84	1	1	0	1
Tiwari	1978.0	Barrow	AK	U.S.A.	Biomass	16	16	1	–	1440	–	50.0	–	–	–	–	–
(8) *Combinations*																	
Hopkinson and Day	1980.0	Barataria	LA	U.S.A.	Water	3	3	56	15	240	65	0	0.68	2	1	1	0
Kemp and Mitsch	1979.0	General		U.S.A.	Biomass	5	0	1	0	30	0	36	0.00	0	0	0	0
Larson et Golet	1982.1	Bristol County	MA	U.S.A.	Area	11	11	1	1	2	2	0	0.00	0	0	0	0
Larson and Golet	1982.2	South Kingston	RI	U.S.A.	Area	7	7	1	1	2	2	0	0.00	0	0	0	0
Wang	1978.0	General	GA	U.S.A.	Oxygen	1	1	1	1	1	1	0	0.99	2	1	0	0

PMC, Principal Modeling Component; MCA, number of model components; DCA, number of data components; MSA, number of model spatial units; DSA, number of data spatial units; MTA, = number of model time intervals; DTA, number of data time intervals; PCNL, percent nonlinearity of the model; FIT, coefficient of determination of the model.

FITCALC = 0 if FIT is not calculatable, 1 if FIT is estimated, 2 if FIT is estimated, and (–) if FIT is missing.

CFM, SFM, and TFM, component, space, and time fit modifiers, respectively; they equal 1 if FIT was calculated for that mode and 0 otherwise.

Numbers following decimal points under year refer to multiple models in the same publication.

Estimating the percent nonlinearity (PCNL) of the models was not always easy. We entered a value of 50% if we knew the models were nonlinear but could not disentangle them sufficiently to do a more precise estimate. The maximum PCNL was 80% for a general bottomland hardwood model by Odum and Brown (1975). Linear models are those with PCNL = 0. Almost 40% of the models we reviewed were linear.

As previously noted, calculation of the descriptive accuracy of the models (FIT in Table II) was complicated by poor reportage and missing data problems. The FITCALC modifier in Table II keeps track of whether the fit was not given or calculatable (0), estimated from given information (1), calculated from given information or reported directly (2), or calculatable but missing (–). Additional fit modifier variables are given that note if the fit was calculated for components (CFM = 1), space (SFM = 1), or time (TFM = 1). These additional modifiers were important for the calculation or effectiveness. For example, if FIT was only calculatable for components (CFM = 1, SFM = 0, and TFM = 0) then effectiveness would be the product of CFM, FIT, and the minimum of the data or model component articulation index (from Table III), divided by three (the number of modes). If FIT was calculable for all three modes (CFM = 1, SFM = 1, and TFM = 1), then effectiveness was the average of the products of FIT and the minimum of the data or model articulation index in each mode.

Table III lists the articulation indices calculated using equation 1 for each model, by mode for the model and the data, listed in order of decreasing average articulation. The average articulation index (AAI) is defined as the minimum of the average model or data articulation listed in Table III. Different types of models are recognizable from their model articulation indices. For example linear, static, input–output models — like Ogawa (1977) or Costanza et al. (1983) — have zero model space and time articulation, but generally high component articulation. Spatially articulated models are rare, with the forest models of Botkin et al. (1972) and Phipps (1979) being notable exceptions. Models that are highly articulated in time are fairly common, but matching time articulation in the data is rare. The models of Mitsch (1975), Gardner et al. (1980), Huff and Young (1980) and Walters et al. (1980), are notable exceptions to this rule. Inspection of the order of the models in Table III shows the possible trade-offs between modes to achieve a high overall articulation. The models of Botkin et al. (1972) and Phipps (1979) top this list by combining high component and space articulation. These models simulate the growth of individual trees in forest plots. Costanza et al. (1983) score high purely on component articulation, while Walters et al. (1980) combine component and time articulation.

Table IV is a summary of model characteristics including the degree of nonlinearity, average articulation index, descriptive accuracy index, and

TABLE III

Articulation indices for the freshwater wetland models reviewed in this study

Model reference		Articulation mode							
Author	Date	Component		Space		Time		Average	
		M	D	M	D	M	D	M	D
Botkin et al.	1972.0	19.3548	19.3548	95.2380	95.2380	16.5972	0.0000	43.7300	38.1976
Phipps	1979.1	32.4324	32.4324	46.5526	46.5526	16.5972	0.0000	31.8607	26.3284
Phipps	1979.2	32.4324	32.4324	46.5526	46.5526	16.5972	0.0000	31.8607	26.3284
Phipps	1979.3	32.4324	32.4324	46.5526	46.5526	16.5972	0.0000	31.8607	26.3284
Phipps	1979.4	32.4324	32.4324	46.5526	46.5526	16.5972	0.0000	31.8607	26.3284
Costanza et al.	1983.1	68.5535	68.5535	0.0000	5.1593	0.0000	0.0000	22.8512	24.5709
Walters et al.	1980.1	35.0649	35.0649	0.0000	0.0000	97.3333	26.6862	44.1327	20.5837
Walters et al.	1980.2	35.0649	35.0649	0.0000	0.0000	97.3333	26.6862	44.1327	20.5837
Costanza et al.	1983.3	57.9832	57.9832	0.0000	5.1593	0.0000	0.0000	19.3277	21.0475
Costanza et al.	1983.4	42.5287	42.5287	0.0000	5.1593	0.0000	0.0000	14.1762	15.8960
Odum and Brown	1975.0	40.4762	0.0000	0.0000	95.2380	0.0000	0.0000	13.4921	31.7460
Bazilevich and Tishkov	1982.0	35.0649	35.0649	0.0000	0.0000	0.0000	0.0000	11.6883	11.6883
Jørgensen	1982.0	24.2424	24.2424	0.0000	0.0000	78.4900	3.2882	34.2441	9.1769
Browder	1978.0	15.2542	21.8750	0.0000	1.1662	23.0177	0.0000	12.7573	7.6804
Mitsch	1976.1	28.5714	21.8750	0.0000	0.0400	99.0099	0.5964	42.5271	7.5038
Ewel and Deghi	1978.0	21.8750	21.8750	0.0000	0.0200	90.9083	0.0000	37.5944	7.2983
Ogawa	1977.0	21.8750	21.8750	0.0000	0.0000	0.0000	0.0000	7.2917	7.2917
Gardner et al.	1980.0	1.9608	1.9608	0.0000	0.0000	19.5495	19.5495	7.1701	7.1701
Mitsch	1976.0	0.0000	0.0000	0.0000	0.0000	21.2356	21.1356	7.0452	7.0452
Huff and Young	1980.0	0.0000	0.0000	0.0000	0.0000	19.2897	19.2897	6.4299	6.4299
Paschal et al.	1979.0	16.6667	16.6667	0.0000	1.9416	2.4390	0.2991	6.3686	6.3024
Larson and Golet	1982.1	16.6667	16.6667	0.0000	0.0000	0.099	0.099	5.5889	5.5889
Williams	1971.1	3.8462	16.6667	0.0000	0.0000	99.0099	0.0000	34.2853	5.5556
Williams	1971.2	15.2542	15.2542	0.0000	0.0000	99.0099	0.0000	38.0880	5.0847
Williams	1971.3	15.2542	15.2542	0.0000	0.0000	99.0089	0.0000	38.0880	5.0847
Williams	1971.4	15.2542	15.2542	0.0000	0.0000	99.0099	0.0000	38.0880	5.0847

Miller et al.	1976.0	13.7931	5.6604	0.0000	0.0000	22.5407	9.0082	12.1113	4.8895
Stone and McHugh	1979.0	3.8462	3.8462	37.6869	0.0799	99.0099	10.6345	46.8477	4.8535
Huff et al.	1973.0	12.2807	12.2807	0.0000	0.0000	63.6231	2.2483	25.3013	4.8430
Kuenzler et al.	1980.2	13.7931	13.7931	0.0000	0.0000	0.0000	0.0000	4.5977	4.5977
Verhagen	1978.0	12.2807	12.2807	0.0000	0.0000	90.9074	0.4975	34.3960	4.2594
Rykiel	1977a.2	12.2807	12.2807	0.0000	0.0000	0.0000	0.0000	4.0936	4.0936
Brown	1978.1	5.6604	1.9608	0.0000	0.0000	95.7445	9.6658	33.8016	3.8755
Nyholm	1978.1	10.7143	10.7143	0.0000	0.0000	26.4165	0.7937	12.3769	3.8360
Sklar	1983.1	10.7143	9.0909	0.0000	0.0000	87.8034	2.2483	32.8392	3.7797
Richey	1977.0	13.7931	10.7143	0.0799	0.0999	0.4975	0.4975	4.7902	3.7706
Mitsch	1983.0	12.2807	10.7143	0.0000	0.0600	90.9074	0.4975	34.3960	3.7573
Ondok and Pokorny	1982.0	5.6604	3.8462	0.0000	0.0000	27.6934	7.3216	11.1179	3.7226
Cleveland et al.	1981.0	5.6604	5.6604	0.0000	5.1593	9.0082	0.1996	4.8895	3.6731
Larson and Golet	1982.2	10.7143	10.7143	0.0000	0.0000	0.0999	0.0999	3.6047	3.6047
Burns and Taylor	1979.1	10.7143	10.7143	0.0000	0.0000	99.0099	0.0000	36.5747	3.5714
Costanza	1975.0	10.7143	10.7143	0.0000	95.2380	0.0000	0.0000	3.5714	35.3174
Costanza	1975.1	10.7143	10.7143	0.0000	95.2380	0.0000	0.0000	3.5714	35.3174
Kuenzler et al.	1980.1	10.7143	10.7143	0.0000	0.0000	0.0000	0.0000	3.5714	3.5714
Sklar	1983.2	10.7143	9.0909	0.0000	0.0000	0.0000	2.2483	3.5714	3.7797
Sklar	1983.3	10.7143	9.0909	0.0000	0.0000	0.0000	2.2483	3.5714	3.7797
Hopkinson and Day	1980.0	3.8462	3.8462	1.0880	0.2792	19.2897	6.0150	8.0746	3.3801
Brown	1978.2	15.2542	7.4074	0.0000	0.0000	0.0000	2.2483	5.0847	3.2186
Wiegert	1971.1	9.0909	9.0909	0.0000	0.0400	26.4165	0.0000	11.8358	3.0436
Wiegert	1971.2	9.0909	9.0909	0.0000	0.0400	26.4165	0.0000	11.8358	3.0436
Wiegert	1971.3	9.0909	9.0909	0.0000	0.0400	26.4165	0.0000	11.8358	3.0436
Wiegert	1971.4	5.6604	9.0909	0.0000	0.0400	26.4165	0.0000	10.6923	3.0436
Wiegert	1971.5	1.9608	9.0909	0.0000	0.0400	26.4165	0.0000	9.4591	3.0436
Wheeler et al.	1978.0	9.0909	9.0909	0.0000	0.0000	4.4890	0.0000	4.5266	3.0303
Mitsch	1975.0	7.4074	7.4074	0.0000	0.0600	99.0099	0.4975	35.4724	2.6550
Brown	1978.3	13.7931	5.6604	0.0000	0.0000	0.0000	2.2483	4.5977	2.6362
Flebbe	1982.1	7.4074	7.4074	0.00000	0.00000	0.0000	0.00000	2.4691	2.46914

TABLE III (*continued*)

Articulation indices for the freshwater wetland models reviewed in this study

Model reference		Articulation mode							
Author	Date	Component		Space		Time		Average	
		M	D	M	D	M	D	M	D
Flebbe	1982.2	7.4074	7.4074	0.00000	0.00000	0.0000	0.00000	2.4691	2.46914
Flebbe	1982.3	7.4074	7.4074	0.00000	0.00000	0.0000	0.00000	2.4691	2.46914
Florschutz	1978.0	5.6604	5.6604	0.00000	0.00000	99.0099	0.00000	34.8901	1.88679
Patten and Matis	1982.0	5.6604	5.6604	0.00000	0.00000	0.0000	0.00000	1.8868	1.88679
Costanza et al.	1983.2	23.0769	0.0000	0.00000	5.15933	0.0000	0.00000	7.6923	1.71978
Costanza et al.	1983.5	10.6349	0.0000	0.00000	5.15933	0.0000	0.00000	6.8783	1.71978
Costanza et al.	1983.6	21.8750	0.0000	0.00000	4.97910	0.0000	0.00000	7.2917	1.65970
Halfon	1979.0	5.6604	1.9608	0.00000	0.00000	23.0177	2.53411	9.5594	1.49830
White et al.	1978.0	0.0000	0.0000	0.59642	0.0990	78.2561	2.62902	26.2842	0.90964
Sklar	1983.0	0.0000	0.0000	0.00000	0.05996	43.4069	2.24829	14.4690	0.76942
Bayley and Odum	1976.0	5.6604	1.9608	0.00000	0.00000	99.0099	0.00000	34.8901	0.65359
Rykiel	1977b.1	1.9608	1.9608	0.00000	0.00000	0.0000	0.00000	0.6536	0.65359

Wiemhoff	1977.0	1.9608	1.9608	0.00000	0.00000	9.3382	0.00000	3.7663	0.65359
Burns and Taylor	1979.0	0.0000	0.0000	2.61005	0.00000	99.0099	0.00000	33.8733	0.00000
Dauffenbach et al.	1981.0	50.0000	0.0000	0.00000	0.00000	–	0.00000	–	0.00000
Douce	1978.0	7.4074	7.4074	0.00000	–	99.0099	–	35.4724	–
Jolánkai	1982.1	0.0000	0.0000	0.00000	0.37856	0.0000	5.57129	0.0000	1.98329
Jolánkai	1982.2	0.0000	0.0000	0.00000	0.37856	0.0000	5.57129	0.0000	1.98329
Kemp and Mitsch	1979.0	7.4074	0.0000	0.00000	0.00000	2.8183	0.00000	3.4086	0.00000
Littlejohn	1977.0	1.9608	1.9608	0.00000	0.21952	99.0099	–	33.6569	–
Loucks and Weiler	1979.0	25.3731	23.0769	0.00000	–	–	–	–	–
Loucks and Weiler	1979.0	25.3731	23.0769	0.00000	–	–	–	–	–
Nessel	1978.0	20.6349	0.0000	0.00000	0.00000	0.0000	0.00000	6.8783	0.00000
Silvola and Hanski	1979.0	0.0000	0.0000	0.00000	0.00000	6.6293	0.00000	2.2098	0.00000
Svirezhev and Voinov	1982.0	5.6604	0.0000	0.00000	0.00000	99.0099	0.00000	34.8901	0.00000
Tiwari et al.	1978.0	23.0769	23.0769	0.00000	–	58.9996	–	27.3588	–
Uhlmann and Recknagel	1982.0	0.0000	0.0000	0.00000	0.21952	0.0000	3.75361	0.0000	1.32438
Walters	1980.0	16.6667	0.0000	0.00000	0.00000	35.0227	0.00000	17.2298	0.000000

The indices for both the models (M) and the data (D) are given for each mode, along with the average over the three modes. The models are listed in decending order starting with the largest average articulation (the minimum of the model and the data).

TABLE IV

Summary of model characteristics

Model reference		PCNL	Articulation	Descriptive accuracy	Effectiveness
Author	Date				
Walters et al.	1980.2	50.0	20.5837	0.253333	7.82181
Gardner et al.	1980.0	50.0	7.1701	0.593333	6.38138
Jørgensen	1982.0	35.8	9.1769	0.460000	6.33204
Huff and Young	1980.0	0.0	6.4299	0.286667	5.52973
Mitsch	1976.1	47.8	7.5038	0.466667	5.24333
Miller et al.	1976.0	50.0	4.8895	0.560000	4.10720
Brown	1978.1	41.6	3.8755	0.626667	3.64298
Ondok and Pokorny	1982.0	14.3	3.7226	0.586667	3.27587
Burns and Taylor	1979.1	8.0	3.5714	0.293333	3.14286
Wheeler et al.	1978.0	0.0	3.0303	0.273333	2.48485
Wiegert	1971.3	50.0	3.0436	0.260000	2.36364
Wiegert	1971.2	50.0	3.0436	0.226667	2.06061
Walters et al.	1980.1	50.0	20.5837	0.066667	2.05837
Botkin et al.	1972.0	20.0	38.1976	0.100000	1.93548
Richey	1977.0	33.3	3.7706	0.333333	1.79904
Sklar	1983.1	45.2	3.7797	0.266667	1.51189
Halfon	1979.0	0.0	1.4983	0.633333	1.42338
Hopkinson and Day	1980.0	0.0	3.3801	0.453333	0.93508
Paschal et al.	1979.0	50.0	6.3024	0.106667	0.90484
Stone and McHugh	1979.0	0.0	4.8535	0.230000	0.88462
Nyholm	1978.1	50.0	3.8360	0.153333	0.88228
Huff et al.	1973.0	50.0	4.8430	0.246667	0.55458
White et al.	1978.0	0.0	0.9096	0.386667	0.52759
Wiegert	1971.5	50.0	3.0436	0.230000	0.45098
Sklar	1983.0	45.2	0.7694	0.333333	0.37471
Wiegert	1971.4	50.0	3.0436	0.063333	0.35849

effectiveness index. Table IV indicates where the most effective modeling efforts for freshwater wetlands have been, both in terms of model types and the level of articulation of the models, since the models are listed in order of decreasing effectiveness. Only 26 of the 87 models we reviewed were complete enough to calculate accuracy and effectiveness, and only these models are listed in Table IV. Among these, it is interesting to note that the highest effectiveness scores were not for the models with the highest articulation or the highest accuracy, nor were they exclusively linear or nonlinear. The most effective models were those that achieved a balance between articulation and accuracy. Note also that the effectiveness index shown in Table IV is not simply the product of average articulation and descriptive accuracy indices shown in Table IV. It is rather the average of the products of FIT and the minimum of model or data articulation for the three modes as described above. Had we used the product of the averages instead, the results would not have been substantially different, however, although the order of some of the models would have changed.

DISCUSSION

Science can be viewed as the process of building successively 'better' descriptive and predictive models of the world. But how does one define 'better'? In the past, scientists have tended to narrow their questions in order to achieve higher accuracy. This leads to models with low articulation but high descriptive accuracy. They say much about little. More recently, scientists have begun to take a 'systems view' that looks at phenomena more comprehensively. This strategy leads to highly articulated models with low accuracy. These models say little about much.

The real effectiveness or explanatory power of a model is a function of *both* how much it attempts to explain (articulation) and how well it explains what was attempted (descriptive accuracy). The overall effectiveness index captures both of these attributes.

The 26 models for which we could calculate accuracy and effectiveness exhibit some interesting patterns in the relationship between articulation and

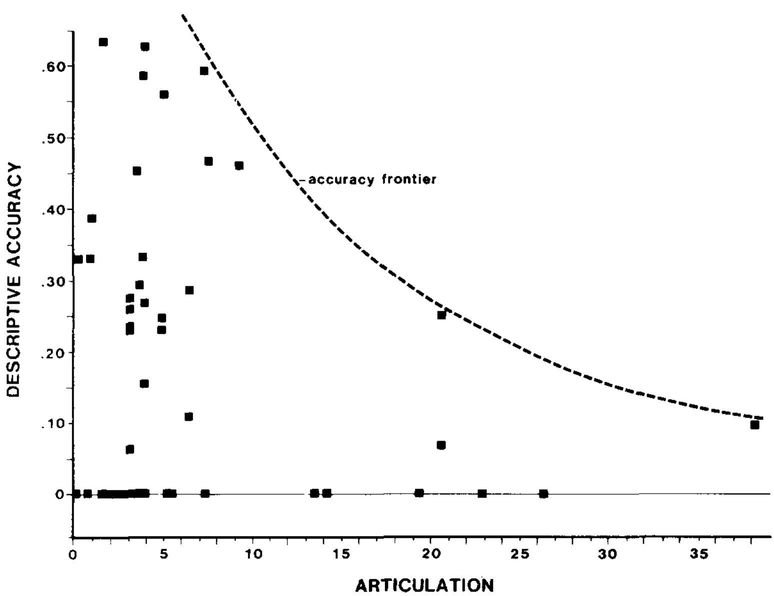

Fig. 1. Plot of articulation index vs. descriptive accuracy index for the models reviewed in this study, showing the current accuracy frontier.

these variables. Figure 1 is a plot of average articulation on the x axis vs. average descriptive accuracy on the y axis. It shows that the maximum accuracy tends to decrease with increasing articulation. This relationship is analogous to the relationship between thermodynamic efficiency and the rate of energy transformation processes (Odum and Pinkerton, 1955; Gutkowicz-Krusin et al., 1978). Efficiency is analogous to accuracy in that both are performance ratios. The thermodynamic efficiency is maximal when the rate of the energy transformation is zero (the reversible or Carnot limit) and decreases as the rate increases. Articulation is analogous to the reaction rate, and accuracy may decrease as articulation increases for reasons analogous to those that cause efficiency to decrease with reaction rate. As the thermodynamic rate increases, entropy and disorder increase and efficiency drops. Likewise, as articulation increases there are more sources of error introduced and maximum accuracy is lowered. It is important to note that the points plotted in Fig. 1 do not all fall on a particular line, but rather below an upper bound line that can be thought of as an 'accuracy frontier'. Many of the models are below the accuracy frontier (the upper bound) for their degree of articulation because they have not been 'completed' sufficiently. It may be possible to push the frontier farther out with additional effort (and cost), but we hypothesize that there is a fundamental limit to this process, such that additional articulation can only be achieved at some cost in accuracy.

Figure 2 is a plot of the articulation index on the x axis vs. model effectiveness on the y axis. The shape of this plot is determined by the shape of the accuracy vs. articulation plot and the definition of effectiveness. The upper bound line of maximum achieved effectiveness (the effectiveness frontier) in Fig. 2 is low for low articulation/high accuracy models (those that say much about little), increases to a maximum for articulation values around 25, and decreases again for high articulation/low accuracy models (those that say little about much). This implies that there is an optimum articulation for maximum model effectiveness at a point substantially below the maximum articulation.

It is important to note that this interpretation depends in large part on the scores for the one model we reviewed with high articulation (Botkin et al., 1972). It therefore cannot be stated with much force until additional data are collected. In informal conversations with ecosystem modelers on this point, however, several unpublished, high articulation models that tend to fit the pattern were mentioned. At this point it should best be considered a hypothesis with a small amount of supporting data.

If it proves accurate, the implications of Fig. 2 for model builders are obvious. There is an optimum size or complexity or articulation as we have called it beyond which the 'benefits' of additional articulation are out-

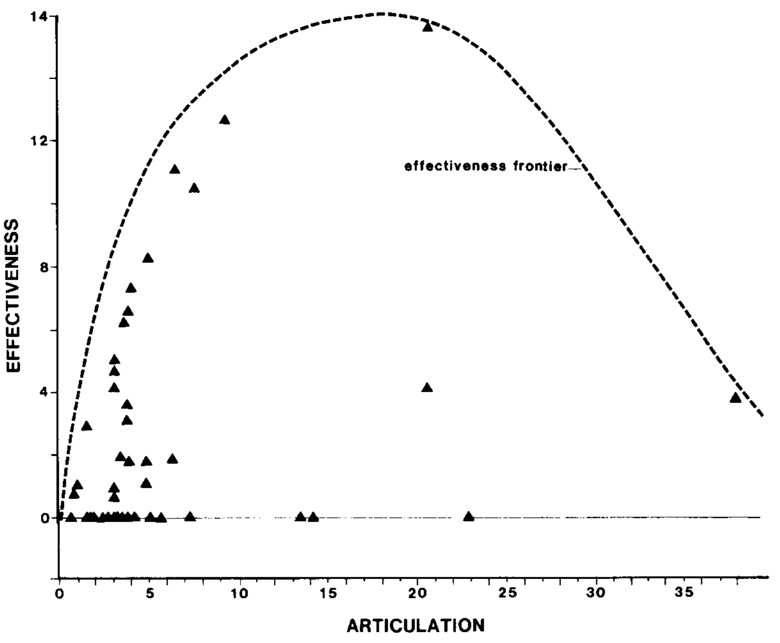

Fig. 2. Plot of articulation index vs. effectiveness index showing the current effectiveness frontier.

weighted by the 'costs' of lowered accuracy. If additional studies are supportive, it could be very useful for model builders and consumers of model output. All types of models are expensive, and ecosystem models are among the most expensive. The question of how complex or articulate to make a model is of primary concern in any modeling study. A reliable guide to solving this problem would help to wisely use the limited resources that can be devoted to ecosystem modeling.

In a more general sense, this result speaks to a fundamental question in science concerning how one should rank alternative explanations. In the past both maximum accuracy and maximum articulation (under different names) have been used as ranking criteria, without much discussion of their potential limits or possible trade-offs. For example, the model of science advocated by Popper (1959, 1972) assumes that anything less than 100% accuracy 'falsifies' a model, and that science achieves 'truth' by successively identifying and discarding 'false' models. We believe the situation is not nearly so black-and-white in most cases, as our modeling review has demonstrated. We must speak of *degrees* of accuracy and articulation and the trade-offs between them, rather than in the absolute terms of truth and falsehood when evaluating complex models. We hope that this paper stimulates more discussion on these trade-offs, and we propose 'model effectiveness' as one way to begin quantifying them and ranking models in a more realistic way.

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