Logical Interrelations between Four Sustainability Parameters: Stability, Continuation, Longevity, and Health

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ABSTRACT

This paper investigates the logical interrelations between four properties that may be useful in understanding different aspects of sustainability and making it a more operational and useful concept. These properties are system stability, continuation, longevity, and health (integrity). The principal findings are as follows: (1) Stability is necessary but not sufficient for sustainability, continuation, longevity, and health. (2) Continuation is (a) sufficient but not necessary for sustainability, stability, and health and (b) both sufficient and necessary for longevity. (3) Longevity is (a) sufficient and necessary for sustainability and continuation and (b) sufficient but not necessary for stability and health. (4) Health is (a) necessary but not sufficient for sustainability, continuation, and longevity and (b) sufficient but not necessary for stability. (5) Sustainability itself is (a) sufficient but not necessary for stability and health; (b) necessary but not sufficient for continuation; and (c) both sufficient and necessary for longevity. These logical interrelations indicate that sustainability is not a simple concept but is related to others, some of which may provide useful measures for different applications. It seems important to explore formally the many dimensions of sustainability to build a precise concept for scientific use. The four attributes investigated here do not exhaust the possibilities.

INTRODUCTION

Sustainability is an important concept with wide interest and a large literature. To make it more useful, its different aspects should be explored and interrelated. Earlier, we discussed the problem of defining the concept and making it more operational (Costanza & Patten 1995). We separated the definition of sustainability from other issues: (1) Which systems or characteristics are to be sustained? (2) For how long? and (3) How do we recognize it when we see it? We argued that because sustainability can be assessed only after the fact, its determination is more a prediction problem than one of definition. We suggested that for evolutionary adaptation to occur there must be an ordered hierarchical relationship between the expected life spans of systems and subsystems as well as the space and time scales of both. Later, Patten (1997) used conditional logic to examine within- and across-scale aspects of three sustainability parameters: stability, continuation, and longevity. Here, a fourth property is added, system health or integrity, and the logic connecting all four within one level of organization is explored. The idea is that by clarifying logical relations between different aspects of sustainability, a less ambiguous and more useful concept may emerge.

CONDITIONAL LOGIC

A conditional statement, or proposition, is of the form "if A then B." A is sufficient for B, and B is necessary for A. The statement is also written "A implies B," or using "$\Rightarrow$" for "implies," "A $\Rightarrow$ B." This is the form of mathematical theorems. A statement may be true or false.

The converse of A $\Rightarrow$ B is "if B then A," or "B $\Rightarrow$ A." Here, B is sufficient for A and A necessary for B. If a statement and its converse are both true, then A and B are each said to be sufficient and necessary for the other, often written "A iff B" (if and only if) or "A $\Leftrightarrow$ B." Letting "$\sim$" denote the negative or complement of a statement, the inverse of A $\Rightarrow$ B is "$\sim$A $\Rightarrow$ $\sim$B," and the contrapositive is "$\sim$B $\Rightarrow$ $\sim$A." The converse and inverse of a statement have the same truth value (true or false); they are logical equivalents. Similarly, the statement and its contrapositive also have the same truth value and are logically equivalent. Mathematicians can prove a theorem by proving its contrapositive and can show necessary conditions by proving the inverse rather than the converse. We can illustrate with an example.

ECOLOGY'S AWFUL THEOREM

What Patten (1997) called ecology's "AWFUL Theorem" operates between competing species in a zero-sum world of limited resources. Physical resources break down into energy and matter categories. Both are conserved as given by mass–energy conservation in physics. The photons that carry electromagnetic energy are not conserved, however, but are multiplied as high-energy photons emitted from a 6000 K solar surface are converted to lower energy infrared photons emitted from the earth's 300 K surface. Twenty (6000 K/300 K) infrared photons are generated to deep space (3 K) for every solar photon received by the planet. This increases the number and random dispersion of particles in the universe and contributes to its overall increase in entropy. Meanwhile, the influx of new solar photons keeps the earth's dynamic systems far from thermodynamic equilibrium (the maximum entropy state). Both natural and human processes take place and interact within this framework.

Soulé (1991) has shown an inverse correlation between the exponential growth of human population over evolutionary time and loss of biodiversity. The AWFUL Theorem (Patten 1997) directed from humans ("we") to other species ("you") over this period is:

$$\underline{A}\text{s } \underline{W}\text{e } \underline{F}\text{lourish yo}\underline{U}\text{ } \underline{L}\text{ose.}$$

Expressed as a logical statement, this becomes:

$$\text{WF} \Rightarrow \text{UL},$$

meaning if we flourish (WF) then you lose (UL), or WF implies ($\Rightarrow$) UL. Human success on the left is sufficient for species failure on the right; moreover, the latter is necessary for the former. The four associated logical forms are:

Theorem:	Converse:
WF $\Rightarrow$ UL	UL $\Rightarrow$ WF
Inverse:	Contrapositive:
$\sim$WF $\Rightarrow$ $\sim$UL	$\sim$UL $\Rightarrow$ $\sim$WF

In each case, sufficient conditions are to the left and necessary ones to the right. The theorem and contrapositive, and the converse and inverse, being logically equivalent pairs, have the same truth value. To establish sufficient conditions either the theorem or its contrapositive must be shown true, and for necessary conditions either the converse or inverse must be proved. To establish both sufficient and necessary conditions, theorem or contrapositive, and converse or inverse, must be shown true.

The AWFUL Theorem itself is generally true in a zero-sum world. Its converse is generally false because there are too many ways for other species to succeed or fail outside the domain of human influence. The inverse is also therefore false; if humanity doesn't flourish, that in itself is no guarantee that other species will. The contrapositive is true because the theorem is true, and that unfortunately is the most insidious form. It says, "If you don't lose we don't flourish!" The AWFUL Theorem is nasty indeed, directed as it always is from winners to losers in the competitive game of life.

A humanity with pretensions to sustainability needs to beware, and the sufficient conditions of theorem and contrapositive are the ones to watch. Fossil and archaeological records amply document both failed species and civilizations as commonplace (Tainter 1988; Yoffee & Cowgill 1988; Ponting 1991). It has been estimated that most (99%) of the species ever to have inhabited the globe have gone extinct. Technological transformation of the planet into habitats and conditions less suitable for man (WL) and more favorable for other species (UF) that can better tolerate global change, pollution, habitat destruction, and depauperization of land- and seascapes—all this could turn the AWFUL Theorem around and aim it back toward man in a new form, UF $\Rightarrow$ WL—or at its contrapositive worst, $\sim$WL $\Rightarrow$ $\sim$UF. This version of the AWFUL Theorem expressed over evolutionary time would gradually foreclose the human episode as the ultimate in unwanted consequences of unsustainability: "As others flourish, . . . we perish."

With this introduction, we can now use propositional logic to investigate some properties related to sustainability that make it more concrete, operational, and ultimately useful. However, the following qualification should be noted. Our decisions about necessary and sufficient conditions do not relate to any specific applied problems. Concepts, definitions, and perspectives often change with applications; therefore, our judgments should only be considered as illustrative of an approach that is best pursued with specific sustainability problems in mind. We consider four properties: stability, continuation, longevity, and health.

STABILITY

Stability has a well-defined mathematical meaning in physical theory, and little adaptation is needed to adjust those concepts to ecological applications (e.g., May 1974; Svirezhev & Logofet 1983; Logofet 1992). Many independently derived ecological concepts had distinct physical science counterparts. Webster *et al.*'s (1975) "resistance" to disturbance resembles bounded or *Lagrange stability*, and their concept of "resilience," the ability to recover to former states after disturbance, corresponds to *Liapunov stability*. Another way of thinking about resilience is to focus on ecosystem dynamics where there are multiple (locally) stable steady states (Holling 1973, 1978, 1986, 1987). Resilience in this sense is a measure of the magnitude of disturbances that can be absorbed before a system centered on one locally stable steady state changes to another. The ecological concepts are intuitive; the mathematical ones are more rigorous and formal.

In many situations, stability might be considered a necessary condition for sustainability:

$$\text{sustainability} \Rightarrow \text{stability}. \qquad (1A)$$

This expression, in its contrapositive form, means that in the absence of stability there can be no sustainability:

$$\sim\text{stability} \Rightarrow \sim\text{sustainability}. \qquad (1B)$$

If, in response to a disturbance a system's trajectory deviated beyond certain limits, this would violate the Lagrange property and negate sustainability. Equally, if the system stayed within bounds but after relaxing the disturbance it failed to return to the vicinity of its former operating states, this violation of the Liapunov criterion would also denote unsustainability. If stability were sufficient for sustainability,

$$\text{stability} \Rightarrow \text{sustainability} \qquad (2A)$$
$$\sim\text{sustainability} \Rightarrow \sim\text{stability} \qquad (2B)$$

(we think this is not true because a stable system might still not endure for very long; see below), then the two concepts would become interchangeable and the well-developed formalism of stability theory could be brought to bear on the assessment of sustainability.

CONTINUATION

Continuation through time is another aspect of sustainability. By this property, we do not mean that a system is "continuous" in the mathematical sense but rather that it satisfies the "continuation" property of system theory (Zadeh & Desoer 1963; Zadeh 1969). This means that future states unfold in a prescribed manner from present (initial) states and that these are arrived at in the same prescribed manner through a succession of past states. This according to Zadeh, is one of five elementary properties needed for a system to have determinate dynamics, that is, to behave such that a unique combination of states and inputs always yields a unique response or output. Nondeterministic dynamics, having stochastic, fuzzy, chaotic, catastrophic, or other forms of uncertainty, are still determinate (as opposed to deterministic) so long as the behaviors are lawful, that is, describable by specifiable parameters. With this small change, we can extend Newtonian determinism to all forms of dynamics whose rules are prescribable, even if the specific manifestations of these rules are not completely predictable. We see no need to deconstruct Newton by insisting that uncertain forms of behavior represent departures from determinism, particularly because much uncertainty is due to the inability to observe fine scales of causation. If one opened a bottle of ether at the front of a room and asked those present to respond when they first smelled it, a wave of response would spread from front to rear. The millions of ether molecules involved would have followed millions of specific Newtonian paths to nostrils, but the process is described stochastically because neither the molecules nor their paths can be individually known. The success of the probabilistic description does not signify that the world is non-Newtonian. Following Zadeh, systems are causal if they are determinate and nonanticipatory; the latter property means that the causes (inputs) do not arrive in time before their effects (outputs) are generated.

The meaning of continuation as we employ it in connection with sustainability is as follows. Let x be a vector of state variables required for a system-level "state-space description." Let $x_{(t_0,t']}$ and $_{(t',t]}$ be time functions from excluded initial times, "$(t_0$" to included final times, and "t]" specifying system behavior in terms of the temporal dynamics of the state variables. Behavior on the interval $(t_0,t]$ is then $x_{(t_0,t]} = x_{(t_0,t']} \cdot x_{(t',t]}$, where "·" denotes concatenation. The trajectory $x_{(t_0,t']}$ and its continuation $x_{(t',t]}$ are taken from a trajectory set X, called the "state space." Let X* be a subset of X such that to the extent that continuations of $x_{(t_0,t']}$ are drawn from $X - X^*$, the system will be understood to be "continued" in time. This means that its future states, $x_{(t',t]}$, will continue to resemble those of the recent past if $x_{(t',t]} \in X - X^*$ also. To the extent that $x_{(t',t]}$, are drawn from X*, however, future behavior will be considered not to continue past patterns, $x_{(t_0,t']}$, and the system (after t) will diverge in its characteristics from those previously established. In other words, under non-continuation the system either drifts or is perturbed away from its customary behavioral norms, which under the conditions experienced during $(t',t]$ must be considered unsustainable.

Continuation, so defined, is sufficient,

$$\text{continuation} \Rightarrow \text{sustainability} \tag{3A}$$
$$\sim\text{sustainability} \Rightarrow \sim\text{continuation}, \tag{3B}$$

but not necessary,

$$\text{sustainability} \sim\Rightarrow \text{continuation} \tag{4A}$$
$$\sim\text{continuation} \sim\Rightarrow \sim\text{sustainability}, \tag{4B}$$

for system sustainability. The symbol "$\sim\Rightarrow$" means "does not imply."

Note, under state-space theory, that from equations (3 and 4) loss of a continuation regime such as $x_{(t_0,t]} = x_{(t_0,t']} \cdot x_{(t',t]} \in X - X^*$ is an irreversible process unless driving inputs are changed. That is, expression (4A) indicates that continuation does not follow from sustainability alone; it must come from another source. A new set of inputs must be experienced to move deviant trajectoris, $x_{(t',t]} \in X^*$, back to $X - X^*$. The system must in other words evolve or be steered by management inputs into new continuation domains if continuation is to be the basis equation (3A) for its sustainability.

LONGEVITY

A third time-related aspect of sustainability is longevity, how long a system exists in relation to its natural, expected, existence time (Costanza & Patten 1995). When one says a system has achieved sustainability, one has to specify the time span involved. Some might argue that sustainability means "maintenance forever." But nothing lasts forever, not even the universe as a whole. Sustainability thus cannot mean an infinite existence interval or nothing would be sustainable. Instead, we suggest that it means an existence time or longevity that is consistent with the system's time and space scale. For example, we expect a cell in an organism to have a relatively short longevity and the longevity to increase as we move up in time and space scales through the organism, the population of organisms, the biome, and the planet as a whole. But no system (even the universe) is expected to have an infinite longevity. A sustainable system in this context is one that attains its full expected longevity within the nested hierarchy of systems within which it is embedded. We refer to this nested hierarchy of systems and subsystems over a range of time and space scales as the "metasystem." Evolution cannot occur unless there is limited longevity of the component parts so that new alternatives can be selected. To maintain a sustainably evolving metasystem, we hypothesize that a particular relationship between the longevity of component subsystems and their time and space scales may be necessary (Costanza & Patten 1995, figure 1).

More formally, let $[t_0,t_f]$ be the expected (mean) existence time of a system in a set of states corresponding to a given continuation domain, $x_{[t_0,t_f]} \in X - X^*$, and let $[t_0,t]$ be its actual duration in this domain, $x_{[t_0,t]} \in X - X^*$. Then, the following condition for sustainability, which is both sufficient and necessary, can be postulated:

$$\text{longevity } (t \geq t_f) \Leftrightarrow \text{sustainability} \tag{5A}$$
$$\sim\text{sustainability} \Leftrightarrow \sim\text{longevity } (t < t_f). \tag{5B}$$

The double arrows ($\Leftrightarrow$) denote both sufficient and necessary, that is, "if and only if" (iff) conditions. If a system lasts, or can be made to last, as long as or longer than it is expected to (drawing state values from $X - X^*$), then and only then is it sustainable by the longevity criterion. As mentioned above, this sustainability is relative to the appropriate time and space scales for the system.

Living systems are sustainable if they achieve their normal life expectancy, and technology

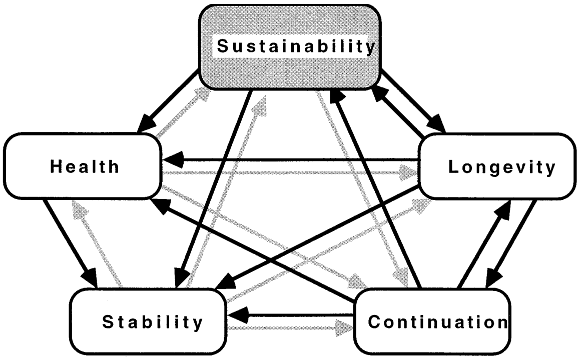

FIGURE 1. Logical interrelations among sustainability and the four related properties of stability, continuation, longevity, and health. The darker arrows denote "implies" ($\Rightarrow$), and the lighter ones denote negation ("does not imply," $\sim\Rightarrow$), as used in text.

(life-support systems) can artificially extend life toward, or even beyond, expected values. Growth and development of organisms, which is transient dynamics toward mature steady states, means that previously established states are not continued in the sense described above. They are not sustainable since ontogenesis cannot be arrested. In populations, mean life expectancy is often used as an indicator of health and well-being. At the level of ecosystems, change occurs through succession as a result of climate change, internal developmental changes, disturbances, and other factors. Ecosystems too have finite existence times, and successional processes represent noncontinuation of prior states within these. In the disturbance realm, eutrophication of aquatic ecosystems causes radical change in the system, ending the existence interval of a more oligotrophic "morph" and initiating that of a more eutrophic one. This is noncontinuation as the oligotrophic states are unsustainable under enrichment; their expected longevity is terminated prematurely by the disturbance conditions. Metasystems with an improper balance of longevity across scales can become either "brittle" when their parts last too long and they cannot adapt fast enough (Holling 1992) or "unsustainable" when their parts do not last long enough and the higher level system's longevity is cut unnecessarily short.

HEALTH (INTEGRITY)

There is considerable current interest in the topic of ecosystem health (Rapport 1989, 1992a, 1992b; Costanza *et al.* 1992). It is generally acknowledged that human health depends on this and that keeping life-support systems "healthy" is essential for a sustainable human future. In simple terms, health reflects overall performance of a complex system resulting from the behavior of its components. Measures of system health imply a weighted sum or more complex operation over the parts, in which the weighting factors incorporate an assessment of the importance of each component to functioning of the whole. This assessment implies "values," which can range from subjective and qualitative to objective and quantitative with increasing knowledge. In medicine, values and weighting factors are implicit in the knowledge and experience of medical practitioners. The concept of health applied to ecosystems is not so clear as for organisms. Ecosystems are more loosely connected entities and, unlike organisms, they have fewer individual examples with which to assess "normal" behavior or perform controlled experiments. Given the complexities involved, the problems of defining ecosystem health or integrity are likely to be just as thorny as those of defining sustainability.

A healthy condition can be considered necessary for the sustainability of living systems (Costanza *et al.* 1992)

$$\text{sustainability} \Rightarrow \text{health} \tag{6A}$$
$$\sim\text{health} \Rightarrow \sim\text{sustainability}, \tag{6B}$$

but health alone is no guarantor of sustainability:

$$\text{health} \sim\Rightarrow \text{sustainability} \tag{7A}$$
$$\sim\text{sustainability} \sim\Rightarrow \sim\text{health}. \tag{7B}$$

If a living system is unhealthy it is unsustainable (6B), but if it unsustainable this does not necessarily mean it is unhealthy (7B). In general, a state of good health is necessary but not sufficient for sustaining life processes.

What seems clear from previous sections and from our prior explorations of sustainability properties (Costanza & Patten 1995; Patten 1997) is that many different attributes are embodied in the sustainability concept. The same is also individually true of these attributes when applied to complex systems—health, longevity, continuation, stability, and probably many others. The difficulties may have less to do with the properties, per se, than with the state of systems knowledge that prevents their precise implementation and measurement. As an example, resolution of the classical diversity–stability problem in ecology (MacArthur 1955; Woodwell & Smith 1969) faltered not so much because diversity and stability could not be defined but because too little was known about the ecosystem complexities to allow these definitions to be implemented. The logical treatment of several parameters of sustainability we have given indicates they are all interrelated, and from this comes the insight that it is the different perspectives deriving from these interrelations that make the concepts of sustainability, health, and others difficult to define and make scientifically operational.

Below, logical interrelations between our four sustainability parameters will be summarized, mindful of the caveat or qualification mentioned above.

LOGICAL INTERRELATIONS

Given the four properties, plus sustainability itself, ten unordered pairings of these are possible. For each, we will show the theorem ($A \Rightarrow B$) and converse ($B \Rightarrow A$) forms that apply at a given level of organization and whether these are true ($\Rightarrow$) or false ($\sim\Rightarrow$):

Sustainability and stability:	sustainability $\Rightarrow$ stability	(1A)
	stability $\sim\Rightarrow$ sustainability	(2A)
Sustainability and continuation:	sustainability $\sim\Rightarrow$ continuation	(4A)
	continuation $\Rightarrow$ sustainability	(3A)
Sustainability and longevity:	sustainability $\Rightarrow$ longevity	(5A)
	longevity $\Rightarrow$ sustainability	(5A)
Sustainability and health:	sustainability $\Rightarrow$ health	(6A)
	health $\sim\Rightarrow$ sustainability	(7A)
Stability and continuation:	stability $\sim\Rightarrow$ continuation	(8)
	continuation $\Rightarrow$ stability	(9)
Stability and longevity:	stability $\sim\Rightarrow$ longevity	(10)
	longevity $\Rightarrow$ stability	(11)
Stability and health:	stability $\sim\Rightarrow$ health	(12)
	health $\Rightarrow$ stability	(13)
Continuation and longevity:	continuation $\Rightarrow$ longevity	(14)
	longevity $\Rightarrow$ continuation	(15)
Continuation and health:	continuation $\Rightarrow$ health	(16)
	health $\sim\Rightarrow$ continuation	(17)
Longevity and health:	longevity $\Rightarrow$ health	(18)
	health $\sim\Rightarrow$ longevity.	(19)

These interrelations are diagrammed in Figure 1.

CONCLUSIONS

The above results indicate the following provisional relations between sustainability and its various properties as applied to a single level of organization. Only longevity is both sufficient and necessary (5A), making this especially important for understanding sustainability (Costanza & Patten 1995). Stability (1A, 2A) and health (6A, 7A) are necessary but not sufficient, whereas continuation is sufficient but not necessary (3A, 4A). As noted, these relationships may change when viewed across scales and also in the context of specific applied problems.

The foregoing analysis shows that (and to some extent, why) sustainability is a complex concept. Stability, continuation, longevity, and health are only four of its related properties. They, and any others, all apply not just at one level of organization ("system") or two ("part" and "whole"; Patten 1997) but actually at multiple scales in the "metasystem" of nature (Costanza & Patten 1995). Because natural systems are complex, as is the concept itself, sustainability has resisted and will continue to resist attempts at comprehensive definition and formulation. This may limit its pragmatic usefulness, but at the same time it underscores the subtleties inherent in the real system and alerts us to the dangers of oversimplification.

There are certainly other properties to be examined by the approach explored here as well as other approaches to take. Each holds potential for new insights and improvement of an evolving operational concept of sustainability. In view of the complexities and the nonlocal holism of the ecosphere, the critical need is to develop a better science of complex systems. Without this, real progress in integrating humans into nature is likely to prove elusive, and the costs of failure could be unbearable if the AWFUL Theorem ever becomes directed back upon humanity. Sustainability for *Homo sapiens* within the metasystem of planet earth requires maintaining a subtle and complex relationship with the other parts of nature at all scales—relationships now being seen as balancing states of "self-organized criticality" (Bak 1996) on the edge between "order and chaos" (Kauffman 1995). Understanding sustainability and particularly how its related properties contribute to it is, we think, necessary (although not sufficient) to achieve this goal.

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